NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1428

NOTES AND TABLES FOR USE IN THE ANALYSIS OF SUPERSONIC FLOW

By The Staff of the Ames 1- by 3-foot
Supersonic Wind-Tunnel Section

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INTRODUCTION

This paper is a compilation of formulas, tables, and curves that has been found to be very useful in the analysis of supersonic wind-tunnel data. The information has been compiled by members of the Ames 1- by 3-foot supersonic wind-tunnel section with the specific needs of supersonic wind-tunnel operation in mind.

With one exception, all tables and curves contained herein have been computed for the value of $\gamma = 1.400$. The one exception is the curves showing the characteristics of cones in supersonic flow. These curves were taken directly from the data of references 1 and 2 wherein a value of $\gamma = 1.405$ was used in the calculations.

Most of the symbols used in the paper are defined in Symbols and Notation. In several sections it has been found necessary to use certain special symbols that would appear only in that particular section, and in such cases those symbols are defined as they are introduced in the text and they are not included in the general list.

Some of the material (e.g., the differential equations of motion) has been included in the information not because it represents material frequently used in wind-tunnel operation, but because it has been found useful as a reference when reading the widely scattered literature on compressible flow.

Explanation of Tables

Three tables included in this paper, Table I–Subsonic, Table II–Supersonic, and Table III–Normal Shock Waves, give numerical values of certain physical quantities that are often used in the calculation of problems involving a compressible fluid. The calculations of these numerical values were carried out to enough places to insure accuracy to four significant figures in the final result. Thus, in most cases six figures were used, but in some cases it was necessary to use eight figures and then round the final result off to four places. A second complete set of independent calculations was carried out and the tables checked in that manner.

The definitions of the symbols used in the tables can be found at the end of tables II and III.

Table I–Subsonic.– The well-known isentropic relations (section I, part B) for the pressure, density, temperature, speed of sound, and area ratios are given as a function of Mach number. If at a point in an isentropic flow, any one of these ratios or the Mach number is known then all other ratios for that point can be read or interpolated from the table. The last three columns are the values of certain parameters (V/a^*, V/a_a, and $V/\hat{V}$) which sometimes are found to be more convenient to use than Mach number. Their relation to Mach number is uniquely determined for any adiabatic flow (section I, part B).

Table II– Supersonic.– In addition to the quantities given in table I others relating the dynamic pressure ($q = \frac{1}{2}\rho V^2$), the Mach angle, and the angle-of-turning through a Prandtl-Meyer expansion are given as a function of Mach number. Logically, this table should also include values of the three parameters mentioned above. The addition of three more columns, however, would make table II too wide to be printed conveniently, so these columns have been included in table III instead. In table III exactly the same values and increments of the Mach number have been used as in table II.

Table III–Normal Shock Waves.– To be consistent with shock-wave notation the Mach number argument in table III has been designated M_o instead of M, hence the three above-mentioned parameters also have subscripts o in table III. This should not be confusing because the relation between Mach number and the corresponding value of V/a^*, V/a , or $V/\hat{V}$ is valid even if shock waves do exist in the flow.

It is to be noted that although the values given in table III are for normal shock waves, many of the columns in the table can also be used for oblique shock waves. (See section IV, part B and section VII, part A.)

SYMBOLS AND NOTATION

a	local velocity of sound
S	area
A	aspect ratio
c	chord of airfoil
c_f	flap chord
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C_D, C_L, C_m	aerodynamic coefficients of drag, lift, and pitching moment, respectively
c_d, c_l, c_m	section force coefficients
d	distance from the leading edge to pitching-moment axis, positive to the rear
e	base of natural logarithms, 2.718...
E	internal energy
h	enthalpy $(pv + E)$
l	characteristic reference length
ln	logarithm to base e
M	Mach number (ratio of local velocity to local velocity of sound)
n	number of degrees of freedom of gas molecule
p	static pressure
P	pressure coefficient $(p-p_o)/q_o$
H	total pressure

q	dynamic pressure $(\frac{1}{2}\rho V^2)$
Q	heat added to system
R	perfect-gas constant
Re	Reynolds number
S	entropy
t	time variable
T	absolute temperature
u,v,w	velocity components
u',v',w'	perturbation velocity components
x,y,z	rectangular coordinates
x,r,θ	cylindrical coordinates
v	specific volume $(1/\rho)$
V	resultant velocity $\left(\sqrt{u^2 + v^2 + w^2}\right)$
$\hat{V}$	maximum velocity obtainable by expanding to zero temperature
W	external work performed
α	angle of attack
α_m	Mach angle $(\sin^{-1} 1/M)$
β	$\sqrt{M^2 - 1}$
γ	ratio of specific heats (c_p/c_v)
δ	angle of deflection of the supersonic stream when passing through an oblique shock wave, or
δ	when prefixed to another symbol, denotes the inexact differential
δ_f	airfoil flap deflection (positive downward)

η	local angle of inclination of airfoil surface with respect to free-stream direction
μ	absolute coefficient of viscosity
φ	perturbation velocity potential
Φ	velocity potential
ρ	mass density
σ	angle between chord line and the tangent to the airfoil surface at a given point
θ	angle between original direction of flow and the shock wave
υ	kinematic viscosity (μ/ρ) or,
υ	angle through which a supersonic stream is turned to expand from $M = 1$ to $M > 1$
ξ	pressure ratio across a shock wave, p_1/p_o

Subscripts

o	refers to free-stream conditions or to conditions just ahead of a shock wave
a	refers to reservoir conditions
1	refers to conditions just behind the initial shock wave or to conditions at a second point of the flow
2	refers to stagnation conditions behind a shock wave
s	refers to conditions on the surface of a cone
u	refers to conditions on upper surface of airfoil
l	refers to conditions on lower surface of airfoil

Superscripts

'	perturbation quantities. (The prime is also used as the symbol for the first derivative in section I-C)

* refers to conditions where $M = 1$

Notation

Notation such as [perf], [isen], etc., that appears after many of the equations signifies that the equation is strictly applicable only if the flow of the fluid complies with the limitations indicated between the brackets. For example,

[perf] means that when applying the equation to compressible flow processes the fluid must be a perfect gas.

[isen perf] means that the flow must take place isentropically and the fluid must be a perfect gas, in order that the equation be applicable.

[adiab] means that the only limitation to the flow process is that no heat be transferred across the streamlines. With this limitation the flow does not necessarily have to take place isentropically, although it may and the equation would of course still be valid.

The restrictions to the equations are intended to indicate the most serious limitations that the flow must comply with. It is desirable to indicate these limitations because shock waves must be dealt with very often in the applications of compressible flow equations, and those equations that are predicated on the assumption of isentropic flow are of course no longer valid. However, many of the equations are predicated only on the less severe assumption of adiabatic flow and such equations are applicable even if shock waves do exist in the flow.

I. FUNDAMENTAL RELATIONSHIPS

A - Thermodynamic Relations

The notation [perf] or [rev] following an equation indicates that it is valid only for a perfect gas or for a reversible process, respectively.

$$c_p \equiv \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{\partial Q}{\partial T}\right)_p = c_v + R \text{ [perf]} = \frac{dh}{dT} \text{ [perf]} \qquad (1)$$

$$c_v \equiv \left(\frac{\partial E}{\partial T}\right)_v = \left(\frac{\partial Q}{\partial T}\right)_v = c_p - R\ [\text{perf}] = \frac{dE}{dT}\ [\text{perf}] \tag{2}$$

$$dE = dQ - dW\ (\text{first law}) = c_v\, dT\ [\text{perf}] \tag{3}$$

$$h \equiv pv + E = c_p T\ [\text{perf}] = (c_v + R)\, T\ [\text{perf}]$$

$$= \frac{\gamma}{\gamma-1}\frac{p}{\rho}\ [\text{perf}] = \frac{a^2}{\gamma-1}\ [\text{perf}] \tag{4}$$

$$dh \equiv pdv + vdp + dE = c_p\, dT\ [\text{perf}] = (c_v + R)\, dT\ [\text{perf}] \tag{5}$$

$$p = \rho RT\ [\text{perf}]\ (\text{perfect gas law}) = \frac{RT}{v}\ [\text{perf}] \tag{6}$$

$$Q = {}_{(\text{rev})}\int TdS$$

$$dQ = dW + dE\ (\text{first law})$$

$$= pdv + dE\ [\text{rev}] = pdv + c_v\, dT\ [\text{perf rev}]$$

$$= dh + vdp\ [\text{rev}] \tag{7}$$

$$R \equiv \frac{p}{\rho T}\ [\text{perf}] \equiv \frac{pv}{T}\ [\text{perf}]$$

$$= c_p - c_v\ [\text{perf}] = c_p\left(\frac{\gamma-1}{\gamma}\right)[\text{perf}]$$

$$= c_v\,(\gamma-1)\ [\text{perf}] \tag{8}$$

$$dS \equiv \left(\frac{dQ}{T}\right)_{rev} = \left(\frac{dE + dW}{T}\right)_{rev} = \frac{dE + pdv}{T} \quad [rev]$$

$$= c_v \frac{dT}{T} + p \frac{dv}{T} \quad [perf\ rev]$$

$$= c_v \frac{dT}{T} - R \frac{d\rho}{\rho} \quad [perf\ rev]$$

$$= c_v \frac{dp}{p} - c_p \frac{d\rho}{\rho} \quad [perf\ rev]$$

$$= c_p \frac{dT}{T} - R \frac{dp}{p} \quad [perf\ rev] \tag{9}$$

$$S = c_v\, ln\, T - R\, ln\, \rho \quad [perf]$$

$$= c_v\, ln\, p - c_p\, ln\, \rho \quad [perf]$$

$$= c_p\, ln\, T - R\, ln\, p \quad [perf]$$

$$= c_v\, ln\, \rho^{-(\gamma-1)}\, T \quad [perf]$$

$$= c_p\, ln\, \rho^{-\frac{\gamma-1}{\gamma}}\, T \quad [perf]$$

$$= R\, ln\, p^{-1}\, T^{\frac{\gamma}{\gamma-1}} \quad [perf] \tag{10}$$

$$\Delta S = {}_{(rev)}\int \frac{dQ}{T} \geqq 0 \ \text{(second law)} \tag{11}$$

$$v \equiv \frac{1}{\rho} = \frac{RT}{p} \quad [perf] \tag{12}$$

$$dv \equiv -\frac{d\rho}{\rho^2} \tag{13}$$

$$dW = dQ - dE \text{ (first law)} = p\,dv \quad [\text{rev}] \tag{14}$$

$$W = \int p\,dv \quad [\text{rev}] \tag{15}$$

$$\gamma \equiv \frac{c_p}{c_v} = \frac{c_v + R}{c_v} \quad [\text{perf}]$$

$$= \frac{n + 2}{n} \quad \text{(kinetic theory)} \tag{16}$$

B — General Equations of Compressible Flow

The notations [perf], [adiab], and [isen] indicate that the equations apply only for a perfect gas, an adiabatic process, or an isentropic process, respectively. An equation without such notation indicates no restrictions.

The fundamental equations of compressible flow along a stream tube are:

Euler's equation:

$$\frac{dp}{\rho} + V\,dV = 0 \tag{17}$$

$$\int \frac{dp}{\rho} + \frac{V^2}{2} = \text{const.} \tag{18}$$

Continuity equation:

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \tag{19}$$

Energy equation:

$$h_0 + \frac{V_0^2}{2} = h_1 + \frac{V_1^2}{2} - Q \tag{20}$$

For adiabatic flow the energy equation becomes

$$h + \frac{V^2}{2} = h_a = \text{const.} \quad [\text{adiab}] \tag{21}$$

$$c_p\, T + \frac{V^2}{2} = c_p\, T_a \qquad [\text{adiab perf}] \tag{22}$$

$$\frac{\gamma}{\gamma-1}\left(\frac{p}{\rho}\right) + \frac{V^2}{2} = \frac{\gamma}{\gamma-1}\left(\frac{p_a}{\rho_a}\right) [\text{adiab perf}] \tag{23}$$

$$\frac{a^2}{\gamma-1} + \frac{V^2}{2} = \frac{a_a{}^2}{\gamma-1} \quad [\text{adiab perf}] \tag{24}$$

From the following relationships

$$\left(\frac{a^*}{a_a}\right)^2 = \frac{2}{\gamma+1} \quad [\text{adiab perf}] \tag{25}$$

$$\left(\frac{a_a}{\hat{V}}\right)^2 = \frac{\gamma-1}{2} \quad [\text{adiab perf}] \tag{26}$$

$$\left(\frac{a^*}{\hat{V}}\right)^2 = \frac{\gamma-1}{\gamma+1} \quad [\text{adiab perf}] \tag{27}$$

the energy equation becomes

$$\frac{a^2}{\gamma-1} + \frac{V^2}{2} = \frac{\hat{V}^2}{2} \quad [\text{adiab perf}] \tag{28}$$

$$\frac{a^2}{\gamma-1} + \frac{V^2}{2} = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right) a^{*2} \, [\text{adiab perf}] \tag{29}$$

<u>Perfect gas law</u>:

$$\frac{p}{\rho} = RT \quad [\text{perf}] \tag{30}$$

Speed of sound equation:

$$a^2 = \frac{\gamma p}{\rho} \ [\text{perf}] = \gamma RT \quad [\text{perf}] \tag{31}$$

Bernoulli's equation:

$$\frac{\gamma}{\gamma-1}\left(\frac{p}{H}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{H}{\rho_a}\right) + \frac{V^2}{2} = \frac{\gamma}{\gamma-1} \quad \frac{H}{\rho_a} \ [\text{isen perf}] \tag{32}$$

$$\left(\frac{V_1}{V_o}\right)^2 = \frac{1-\left(\frac{p_1}{H_o}\right)^{\frac{\gamma-1}{\gamma}}}{1-\left(\frac{p_o}{H_o}\right)^{\frac{\gamma-1}{\gamma}}} \ [\text{isen perf}] \tag{33}$$

Isentropic relations:

$$\frac{p}{\rho^\gamma} = \text{constant} \quad [\text{isen perf}] \tag{34}$$

$$\left(\frac{p}{H}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\rho}{\rho_a}\right)^{\gamma-1} = \left(\frac{a}{a_a}\right)^2 = \left(\frac{T}{T_a}\right) \ [\text{isen perf}] \tag{35}$$

The following equations are derived from the above relationships and are grouped according to which of the various parameters $\left(M, \frac{V}{a^*}, \frac{V}{a_a}, \frac{V}{\hat{V}}\right)$ is being used as the independent variable. The second form of the equations apply to air for which $\gamma=1.400$.

<u>Parameter</u> — $\frac{V}{a} = M$

$$\frac{p}{H} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} = (1 + 0.2\ M^2)^{-\frac{7}{2}} \quad \text{[isen perf]} \qquad (36)$$

$$\frac{\rho}{\rho_a} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{\gamma-1}} = (1 + 0.2\ M^2)^{-\frac{5}{2}} \quad \text{[isen perf]} \qquad (37)$$

$$\frac{T}{T_a} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} = (1 + 0.2\ M^2)^{-1} \quad \text{[adiab perf]}$$

$$\frac{a}{a_a} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{2}} = (1 + 0.2\ M^2)^{-\frac{1}{2}} \quad \text{[adiab perf]} \qquad (39)$$

$$\frac{q}{p} = \frac{\gamma}{2} M^2 = 0.7\ M^2 \quad \text{[perf]} \qquad (40)$$

$$\frac{q}{H} = \frac{\gamma}{2} M^2 \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} = 0.7\ M^2\ (1 + 0.2\ M^2)^{-\frac{7}{2}} \quad \text{[isen perf]} \qquad (41)$$

$$\left(\frac{V}{a_a}\right)^2 = \frac{M^2}{1 + \frac{\gamma-1}{2} M^2} = \frac{M^2}{1 + 0.2\ M^2} \quad \text{[adiab perf]} \qquad (42)$$

$$\left(\frac{V}{a^*}\right)^2 = \frac{\frac{\gamma+1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2} = \frac{1.2\ M^2}{1 + 0.2\ M^2} \quad \text{[adiab perf]} \qquad (43)$$

$$\left(\frac{V}{\hat{V}}\right)^2 = \frac{\frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} M} = \frac{0.2\ M^2}{1 + 0.2\ M^2} \qquad \text{[adiab perf]} \tag{44}$$

$$\frac{p_1}{p_o} = \left(\frac{1 + \frac{\gamma-1}{2} M_o{}^2}{1 + \frac{\gamma-1}{2} M_1{}^2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1 + 0.2\ M_o{}^2}{1 + 0.2\ M_1{}^2}\right)^{\frac{7}{2}} \qquad \text{[isen perf]} \tag{45}$$

$$\left(\frac{V_1}{V_o}\right)^2 = \left(\frac{M_1}{M_o}\right)^2 \left(\frac{1 + \frac{\gamma-1}{2} M_o{}^2}{1 + \frac{\gamma-1}{2} M_1{}^2}\right) = \left(\frac{M_1}{M_o}\right)^2 \left(\frac{1 + 0.2\ M_o{}^2}{1 + 0.2\ M_1{}^2}\right) \qquad \text{[adiab perf]} \tag{46}$$

$$\frac{p}{H} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V}{a^*}\right)^2\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{[isen perf]} \tag{47}$$

$$\frac{\rho}{\rho_a} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V}{a^*}\right)^2\right]^{\frac{1}{\gamma-1}} \qquad \text{[isen perf]} \tag{48}$$

$$\frac{T}{T_a} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V}{a^*}\right)^2\right] \qquad \text{[adiab perf]} \tag{49}$$

$$\frac{a}{a_a} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V}{a^*}\right)^2\right]^{\frac{1}{2}} \qquad \text{[adiab perf]} \tag{50}$$

$$\frac{q}{p} = \frac{\frac{\gamma}{\gamma+1}\left(\frac{V}{a^*}\right)^2}{1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V}{a^*}\right)^2} \qquad \text{[adiab perf]} \tag{51}$$

$$\frac{q}{H} = \frac{\gamma}{\gamma+1}\left(\frac{V}{a^*}\right)^2\left[1 - \frac{\gamma-1}{\rho+1}\left(\frac{V}{a^*}\right)^2\right]^{\frac{1}{\gamma-1}} \text{[isen perf]} \tag{52}$$

$$M^2 = \frac{\frac{2}{\gamma+1}\left(\frac{V}{a^*}\right)^2}{1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V}{a^*}\right)^2} \qquad \text{[adiab perf]} \tag{53}$$

$$\left(\frac{V}{a_a}\right)^2 = \frac{2}{\gamma+1}\left(\frac{V}{a^*}\right)^2 \qquad \text{[adiab perf]} \tag{54}$$

$$\left(\frac{V}{\hat{V}}\right)^2 = \frac{\gamma-1}{\gamma+1}\left(\frac{V}{a^*}\right)^2 \qquad \text{[adiab perf]} \tag{55}$$

$$\frac{p_1}{p_0} = \left[\frac{1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_1}{a^*}\right)^2}{1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_0}{a^*}\right)^2}\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{[isen perf]} \tag{56}$$

$$\left(\frac{V_1}{V_0}\right)^2 = \frac{\left(\frac{V_1}{a^*}\right)^2}{\left(\frac{V_0}{a^*}\right)^2} \tag{57}$$

<u>Parameter</u> – $\frac{V}{a_a}$

$$\frac{p}{H} = \left[1 - \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2\right]^{\frac{\gamma}{\gamma-1}} \text{[isen perf]} \tag{58}$$

$$\frac{\rho}{\rho_a} = \left[1 - \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2\right]^{\frac{1}{\gamma-1}} \qquad \text{[isen perf]} \tag{59}$$

$$\frac{T}{T_a} = \left[1 - \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2\right] \qquad \text{[adiab perf]} \tag{60}$$

$$\frac{a}{a_a} = \left[1 - \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2\right]^{\frac{1}{2}} \qquad \text{[adiab perf]} \tag{61}$$

$$\frac{q}{p} = \frac{\frac{\gamma}{2}\left(\frac{V}{a_a}\right)^2}{1 - \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2} \qquad \text{[adiab perf]} \tag{62}$$

$$\frac{q}{H} = \frac{\gamma}{2}\left(\frac{V}{a_a}\right)^2\left[1 - \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2\right]^{\frac{1}{\gamma-1}} \qquad \text{[isen perf]} \tag{63}$$

$$M^2 = \frac{\left(\frac{V}{a_a}\right)^2}{1 - \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2} \qquad \text{[adiab perf]} \tag{64}$$

$$\left(\frac{V}{a^*}\right)^2 = \frac{\gamma+1}{2}\left(\frac{V}{a_a}\right)^2 \qquad \text{[adiab perf]} \tag{65}$$

$$\left(\frac{V}{\hat{V}}\right)^2 = \frac{\gamma-1}{2}\left(\frac{V}{a_a}\right)^2 \qquad \text{[adiab perf]} \tag{66}$$

$$\frac{p_1}{p_0} = \left[\frac{1 - \frac{\gamma-1}{2}\left(\frac{V_1}{a_a}\right)^2}{1 - \frac{\gamma-1}{2}\left(\frac{V_0}{a_a}\right)^2}\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{isen perf} \tag{67}$$

$$\left(\frac{V_1}{V_0}\right)^2 = \frac{\left(\frac{V_1}{a_a}\right)^2}{\left(\frac{V_0}{a_a}\right)^2} \tag{68}$$

Parameter – $\frac{V}{\hat{V}}$

$$\frac{p}{H} = \left[1 - \left(\frac{V}{\hat{V}}\right)^2\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{[isen perf]} \tag{69}$$

$$\frac{\rho}{\rho_a} = \left[1 - \left(\frac{V}{\hat{V}}\right)^2\right]^{\frac{1}{\gamma-1}} \qquad \text{[isen perf]} \tag{70}$$

$$\frac{T}{T_a} = \left[1 - \left(\frac{V}{\hat{V}}\right)^2\right] \qquad \text{[adiab perf]} \tag{71}$$

$$\frac{a}{a_a} = \left[1 - \left(\frac{V}{\hat{V}}\right)^2\right]^{\frac{1}{2}} \qquad \text{[adiab perf]} \tag{72}$$

$$\frac{q}{p} = \frac{\frac{\gamma}{\gamma-1}\left(\frac{V}{\hat{V}}\right)^2}{1 - \left(\frac{V}{\hat{V}}\right)^2} \qquad \text{[adiab perf]} \tag{73}$$

$$\frac{q}{H} = \frac{\gamma}{\gamma-1}\left(\frac{V}{\hat{V}}\right)^2 \left[1 - \left(\frac{V}{\hat{V}}\right)^2\right]^{\frac{1}{\gamma-1}} \qquad \text{[isen perf]} \tag{74}$$

$$M^2 = \frac{\frac{2}{\gamma-1}\left(\frac{V}{\hat{V}}\right)^2}{1 - \left(\frac{V}{\hat{V}}\right)^2} \qquad \text{[adiab perf]} \tag{75}$$

$$\left(\frac{V}{a_a}\right)^2 = \frac{2}{\gamma-1}\left(\frac{V}{\hat{V}}\right)^2 \qquad \text{[adiab perf]} \tag{76}$$

$$\left(\frac{V}{a^*}\right)^2 = \frac{\gamma+1}{\gamma-1}\left(\frac{V}{\hat{V}}\right)^2 \qquad \text{[adiab perf]} \tag{77}$$

$$\frac{p_1}{p_0} = \left[\frac{1-\left(\frac{V_1}{\hat{V}}\right)^2}{1-\left(\frac{V_0}{\hat{V}}\right)^2}\right]^{\frac{\gamma}{\gamma-1}} \qquad \text{[isen perf]} \tag{78}$$

$$\left(\frac{V_1}{V_0}\right)^2 = \frac{\left(\frac{V_1}{\hat{V}}\right)^2}{\left(\frac{V_0}{\hat{V}}\right)^2} \tag{79}$$

With the Mach number M as a parameter, numerical values will be found in table II for

$$\frac{p}{H}, \frac{\rho}{\rho_a}, \frac{T}{T_a}, \frac{a}{a_a}, \frac{q}{p}, \frac{q}{H}$$

and in table III for

$$\frac{V}{a_a}, \frac{V}{a^*}, \frac{V}{\hat{V}}$$

C – Differential Equations of Motion

Rectangular coordinates (x, y, z).– The basic differential equations of fluid motion with friction and gravity forces neglected are:

1. The equation of continuity

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{80}$$

2. The momentum equations

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial u} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} \tag{81}$$

If pressure is a function only of the density [isen], then

$$\frac{\partial p}{\partial x} = \frac{dp}{d\rho} \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial y} = \frac{dp}{d\rho} \frac{\partial \rho}{\partial y} \qquad \frac{dp}{d\rho} = a^2$$

$$\frac{\partial p}{\partial z} = \frac{dp}{d\rho} \frac{\partial \rho}{\partial z} \tag{82}$$

In the following equations a steady frictionless flow is postulated.

<u>Combinations of basic equations</u>.— Combining equations (80), (81), and (82) yields one nonlinear differential equation

$$\frac{\partial u}{\partial x}\left(1 - \frac{u^2}{a^2}\right) + \frac{\partial v}{\partial y}\left(1 - \frac{v^2}{a^2}\right) + \frac{\partial w}{\partial z}\left(1 - \frac{w^2}{a^2}\right) - \frac{uv}{a^2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$- \frac{vw}{a^2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) - \frac{wu}{a^2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0 \qquad \text{[isen]} \tag{83}$$

For irrotational flow [irrot] the following purely geometric relations hold

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \qquad \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z} \qquad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \qquad \text{[irrot]} \tag{84}$$

also

$$u = \frac{\partial \Phi}{\partial x} \qquad v = \frac{\partial \Phi}{\partial y} \qquad w = \frac{\partial \Phi}{\partial z} \qquad \text{[irrot]} \tag{85}$$

Combining (84), (85), and (83) gives the differential equation for the velocity potential

$$\frac{\partial^2 \Phi}{\partial x^2}\left[1 - \left(\frac{1}{a}\frac{\partial \Phi}{\partial x}\right)^2\right] + \frac{\partial^2 \Phi}{\partial y^2}\left[1 - \left(\frac{1}{a}\frac{\partial \Phi}{\partial y}\right)^2\right] + \frac{\partial^2 \Phi}{\partial z^2}\left[1 - \left(\frac{1}{a}\frac{\partial \Phi}{\partial z}\right)^2\right]$$

$$\frac{2}{\partial x^2}\frac{\partial^2 \Phi}{\partial x \partial y}\frac{\partial \Phi}{\partial x}\frac{\partial \Phi}{\partial y} - \frac{2}{a^2}\frac{\partial^2 \Phi}{\partial y \partial z}\frac{\partial \Phi}{\partial y}\frac{\partial \Phi}{\partial z} - \frac{2}{a^2}\frac{\partial^2 \Phi}{\partial z \partial x}\frac{\partial \Phi}{\partial z}\frac{\partial \Phi}{\partial x} = 0 \quad \text{[isen irrot]} \tag{86}$$

<u>Cylindrical coordinates (x, r, θ)</u>.— The differential equation for the velocity potential is

$$\frac{\partial^2 \Phi}{\partial x^2}\left[1 - \left(\frac{1}{a}\frac{\partial \Phi}{\partial x}\right)^2\right] + \frac{\partial^2 \Phi}{\partial r^2}\left[1 - \left(\frac{1}{a}\frac{\partial \Phi}{\partial r}\right)^2\right] + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}\left[1 - \left(\frac{1}{ra}\frac{\partial \Phi}{\partial \theta}\right)^2\right]$$

$$- \frac{2}{a^2}\frac{\partial^2 \Phi}{\partial x \partial r}\frac{\partial \Phi}{\partial x}\frac{\partial \Phi}{\partial r} - \frac{2}{a^2 r^2}\frac{\partial^2 \Phi}{\partial x \partial \theta}\frac{\partial \Phi}{\partial x}\frac{\partial \Phi}{\partial \theta} - \frac{2}{a^2 r^2}\frac{\partial^2 \Phi}{\partial r \partial \theta}\frac{\partial \Phi}{\partial r}\frac{\partial \Phi}{\partial \theta}$$

$$+ \frac{1}{a^2 r^3}\left(\frac{\partial \Phi}{\partial \theta}\right)^2 \frac{\partial \Phi}{\partial r} + \frac{1}{r}\frac{\partial \Phi}{\partial r} = 0 \qquad \text{[isen irrot]} \tag{87}$$

Cylindrical Coordinates with Axial Symmetry (x, r).– In this system x is measured in the direction of the undisturbed flow and r is measured perpendicular to x. The velocity components u, v are measured in the x and r directions, respectively. The equation of motion is

$$\frac{\partial u}{\partial x}\left(1-\frac{u^2}{a^2}\right)+\frac{\partial v}{\partial r}\left(1-\frac{v^2}{a^2}\right)-\frac{uv}{a^2}\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)+\frac{v}{r}=0 \quad \text{[isen]} \qquad (88)$$

The condition of irrotationality is

$$\frac{\partial u}{\partial r}=\frac{\partial v}{\partial x} \quad \text{[irrot]} \qquad (89)$$

The differential equation for the potential of the irrotational flow is

$$\frac{\partial^2\Phi}{\partial x^2}\left[1-\left(\frac{1}{a}\frac{\partial\Phi}{\partial x}\right)^2\right]+\frac{\partial^2\Phi}{\partial r^2}\left[1-\left(\frac{1}{a}\frac{\partial\Phi}{\partial r}\right)^2\right]-\frac{2}{a^2}\frac{\partial^2\Phi}{\partial x\partial r}\frac{\partial\Phi}{\partial x}\frac{\partial\Phi}{\partial r}+\frac{1}{r}\frac{\partial\Phi}{\partial r}=0$$

$$\text{[isen irrot]} \qquad (90)$$

Linearized Forms.– The perturbations from the undisturbed flow are assumed to be small enough so that their squares and cross products can be neglected. The velocity potential is written in two parts, that is, $\Phi = V_0x + \varphi$. The velocity components are

$$\begin{aligned} u &= V_0 + u' \qquad & u' &= \frac{\partial\varphi}{\partial x} \\ v &= v' & v' &= \frac{\partial\varphi}{\partial y} \\ w &= w' & w' &= \frac{\partial\varphi}{\partial z} \end{aligned} \qquad (91)$$

For two-dimensional flow the linearized differential equation for the perturbation potential is

$$\frac{\partial^2\varphi}{\partial x^2}\left(\frac{V_0{}^2}{a_0{}^2}-1\right)-\frac{\partial^2\varphi}{\partial y^2}=0 \qquad (92)$$

The general solution to equation (92) is

$$\varphi = f_1(x-\beta_1 y) + f_2(x+\beta_1 y) \tag{93}$$

where f_1 and f_2 are arbitrary functions and

$$\beta_1 = \sqrt{M_o^2-1} \qquad M_o = \frac{V_o}{a_o}$$

For the case of axially symmetric supersonic flow (thin bodies of revolution) the linearized form in cylindrical coordinates is

$$(M_o^2-1)\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial r^2} - \frac{1}{r}\frac{\partial\varphi}{\partial r} = 0 \tag{94}$$

The general solution to this equation is

$$\varphi = \int_0^{x-\beta r} f(\xi)\frac{d\xi}{\sqrt{(x-\xi)^2 - \beta_1^2 r^2}}$$

or

$$\varphi = \int_{\cosh^{-1}\frac{x}{\beta r}}^{0} f(x-\beta_1 r \cosh\eta)\, d\eta \tag{95}$$

where

$\beta_1 = \sqrt{M_o^2-1}$

ξ and η are variables of integration, $\dfrac{x-\xi}{\beta_1 r} = \eta$.

$f(\xi)$ is an arbitrary function

Equation (95) can be used to calculate the pressure distribution about a slender body of revolution of arbitrary shape (but pointed nose) at zero angle of attack, by using the following additional equations

$$\Delta p = -\rho V_o u' \tag{96}$$

$$u' = \frac{\partial\varphi}{\partial x} = -\int_0^{x-\beta r} f'(\xi)\frac{d\xi}{\sqrt{(x-\xi)^2 - \beta_1^2 r^2}} \tag{97}$$

$$\frac{f(x)}{V_o} = r\,\frac{dr}{dx} \tag{98}$$

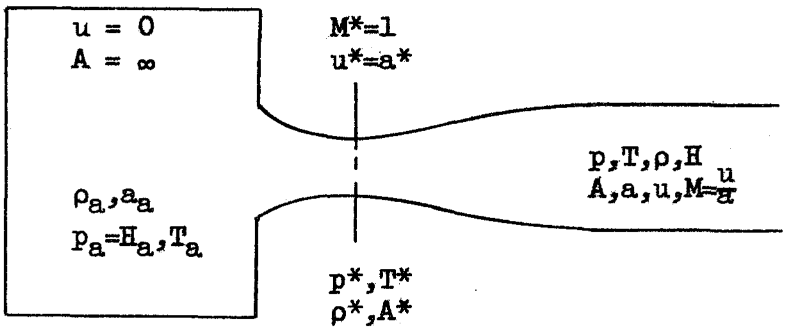
Other sometimes useful relationships are

$$V_o\,\frac{dr}{dx} = \frac{\partial\varphi}{\partial r} = v'$$

$$v' = \frac{1}{r}\int_0^{x-\beta r} \frac{f'(\xi)\,(x-\xi)\,d\xi}{\sqrt{(x-\xi)^2 - \beta_1{}^2 r^2}} \tag{99}$$

II – SUPERSONIC NOZZLES

A – One-Dimensional Theory



The use of the continuity equation in the form

$$\rho uA = \rho^* u^* A^* = \text{constant} \tag{100}$$

requires that the flow be assumed unidimensional, that is, it requires that the velocity profile be straight and the velocity component v be neglected. Then $u \cong V$.

By combining the above equation with suitable equations in section I–B, the following equations for the area ratio are derived

$1+\frac{\gamma-1}{2}(1)^2$

$$\frac{A^*}{A} = M\left(\frac{\frac{\gamma+1}{2}}{1 + \frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{1.728\,M}{(1 + 0.2\,M^2)^3} \quad \text{[isen perf]} \tag{101}$$

$$\frac{A^*}{A} = \left(\frac{u}{a_a}\right)\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[1-\frac{\gamma-1}{2}\left(\frac{u}{a_a}\right)^2\right]^{\frac{1}{\gamma-1}} \quad \text{[isen perf]} \quad (102)$$

$$\frac{A^*}{A} = \left(\frac{u}{a^*}\right)\sqrt{\frac{2}{\gamma+1}}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[1-\frac{\gamma-1}{\gamma+1}\left(\frac{u}{a^*}\right)^2\right]^{\frac{1}{\gamma-1}} \quad \text{[isen perf]} \quad (103)$$

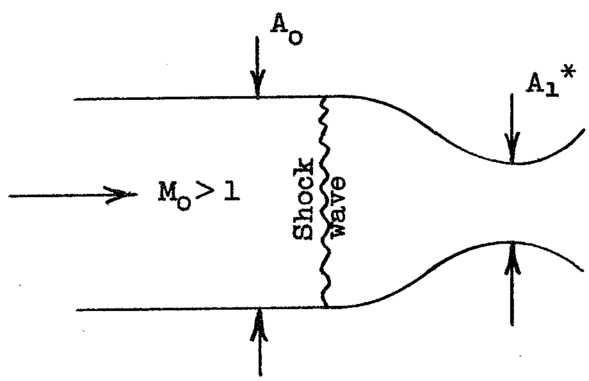
$$\frac{A^*}{A} = \left(\frac{u}{\hat{u}}\right)\sqrt{\frac{2}{\gamma-1}}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[1-\left(\frac{u}{\hat{u}}\right)^2\right]^{\frac{1}{\gamma-1}} \quad \text{[isen perf]} \quad (104)$$

Numerical values for $\frac{A^*}{A}$ will be found in table II as a function of M.

The compressible flow equations of section I and the numerical values in table II are applicable in the unidimensional analysis.

B – Nozzle Data

Maximum theoretical contraction ratio that permits start of supersonic flow in diffuser entrance $\left(\frac{A_o}{A_1^*}\right)_{max}$



The following equation is derived for unidimensional flow which is isentropic upstream and downstream of (but not through) the normal shock wave

$$\left(\frac{A_o}{A_1^*}\right)_{max} = \frac{\left(\frac{\gamma+1}{\gamma-1} M_o\right)^{\frac{\gamma+1}{\gamma-1}} \left(1 + \frac{\gamma-1}{2} M_o^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}{\left(1 + \frac{\gamma-1}{2}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \left(\frac{2}{\gamma-1} + M_o^2\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma-1} M_o^2 - 1\right)^{\frac{1}{\gamma-1}}}$$

$$= \frac{27,000\ M_o^6(1 + 0.2\ M_o^2)^3}{(5 + M_o^2)^{3.5}(7M_o^2 - 1)^{2.5}} \qquad (105)$$

When supersonic flow has been established in the diffuser entrance the normal shock wave stands downstream of the second throat A_1^*.

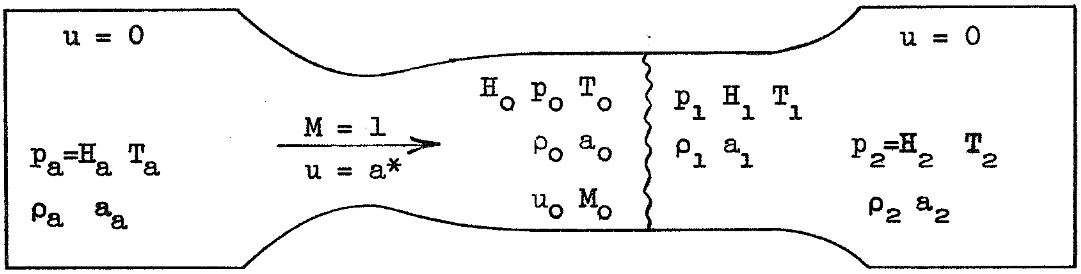
Supersonic diffusers without initial boundary layer check the theory very closely, but if there is an initial boundary layer the maximum contraction ratio is reduced and the equation is a fair first approximation.

Numerical values for $\left(\frac{A_o}{A_1^*}\right)_{max}$ may be picked from the curve of figure 1.

III – SHOCK WAVES

A – Normal Shock Waves

The previous relations for isentropic flow are valid for points on either side of a shock wave (e.g., a and 0, or 1 and 2), but not across it.



The general energy equation

$$\left.\begin{aligned}
&\frac{u^2}{2} + h = h_a && \text{(assuming adiabatic flow)}\\
&\frac{u^2}{2} + c_pT = c_pT_a && \text{(assuming adiabatic flow of a perfect gas)}\\
&\frac{u^2}{2} + \frac{\gamma}{\gamma-1}\,\frac{p}{\rho} = \frac{\gamma}{\gamma-1}\,\frac{H_a}{\rho_a} &&\\
&\frac{u^2}{2} + \frac{p}{\rho} + c_vT = \frac{H_a}{\rho_a} + c_vT_a &&\\
&\frac{u^2}{2} + \frac{a^2}{\gamma-1} = \frac{{a_a}^2}{\gamma-1} = \frac{\gamma+1}{2(\gamma-1)}\,a^{*2} &&
\end{aligned}\right\} \qquad (106)$$

Hence

$$\left.\begin{aligned}
H_o &= H_a\\
H_1 &= H_2\\
T_2 &= T_a\\
a_2 &= a_a
\end{aligned}\right\} \qquad (107)$$

Together with the momentum equation

$$p_o + \rho_o u_o^2 \; p_1 + \rho_1 u_1^2 \tag{108}$$

and the continuity equation

$$\rho_o u_o = \rho_1 u_1 \tag{109}$$

the energy equation provides the following relations across the shock wave

$$u_o u_1 = a^{*2} \qquad \text{(Prandtl's relation)} \tag{110}$$

$$\frac{\rho_o}{\rho_1} = \frac{(\gamma-1)p_1 + (\gamma+1)p_o}{(\gamma-1)p_o + (\gamma+1)p_1} \qquad \text{(Rankine-Hugoniot relation)} \tag{111}$$

<u>Given M_o</u>

($M_o > 1.0$)

$$M_1^2 = \frac{(\gamma-1)\, M_o^2 + 2}{2\gamma M_o^2 - (\gamma-1)} = \frac{M_o^2 + 5}{7M_o^2 - 1} \tag{112}$$

$$\frac{p_1}{p_o} \equiv \xi = \frac{2\gamma M_o^2 - (\gamma-1)}{(\gamma+1)} = \frac{7M_o^2 - 1}{6} \tag{113}$$

$$\frac{\rho_1}{\rho_o} = \frac{u_o}{u_1} = \frac{u_o^2}{a^{*2}} = \frac{(\gamma+1)M_o^2}{(\gamma-1)M_o^2 + 2} = \frac{6M_o^2}{M_o^2 + 5} \tag{114}$$

$$\frac{T_1}{T_o} = \frac{a_1^2}{a_o^2} = \frac{[2\gamma M_o^2 - (\gamma-1)][(\gamma-1)M_o^2 + 2]}{(\gamma+1)^2 M_o^2} = \frac{(7M_o^2 - 1)(M_o^2 + 5)}{36\; M_o^2} \tag{115}$$

$$\frac{p_1}{H} = \frac{2\gamma M_o^2 - (\gamma-1)}{\gamma+1}\left[\frac{(\gamma-1)M_o^2 + 2}{2}\right]^{-\frac{\gamma}{\gamma-1}} = \frac{7M_o^2 - 1}{6}\left(\frac{M_o^2 + 5}{5}\right)^{-\frac{7}{2}} \tag{116}$$

$$\frac{p_1}{H_1} = \left[\frac{4\gamma M_0{}^2 - 2(\gamma-1)}{(\gamma+1)^2 M_0{}^2}\right]^{\frac{\gamma}{\gamma-1}} = \left[\frac{36\ M_0{}^2}{5(7M_0{}^2 - 1)}\right]^{-\frac{7}{2}} \tag{117}$$

$$\frac{H_1}{H_0} = \frac{H_2}{H_a} = \frac{\rho_2}{\rho_a} = e^{-\frac{\Delta S}{R}} = \left[\frac{(\gamma+1)M_0{}^2}{(\gamma-1)M_0{}^2 + 2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2\gamma M_0{}^2 - (\gamma-1)}{\gamma+1}\right]^{-\frac{1}{\gamma-1}}$$

$$= \left(\frac{6M_0{}^2}{M_0{}^2 + 5}\right)^{\frac{7}{2}}\left(\frac{7M_0{}^2 - 1}{6}\right)^{-\frac{5}{2}} \tag{118}$$

$$\frac{H_1}{p} = \left[\frac{(\gamma+1)M_0{}^2}{2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2\gamma M_0{}^2 - (\gamma-1)}{\gamma+1}\right]^{-\frac{1}{\gamma-1}} = \left(\frac{6M_0{}^2}{5}\right)^{\frac{7}{2}}\left(\frac{7M_0{}^2 - 1}{6}\right)^{-\frac{5}{2}}$$

$$\tag{119}$$

$$\frac{\Delta S}{c_v} = (\gamma-1)\ \frac{\Delta S}{R} = -\ (\gamma-1)\ \ln\left(\frac{H_1}{H_0}\right)$$

$$= \ln\left[\frac{2\gamma M_0{}^2 - (\gamma-1)}{\gamma+1}\right] - \gamma\ \ln\left[\frac{(\gamma+1)M_0{}^2}{(\gamma-1)M_0{}^2 + 2}\right] \tag{120}$$

Numerical values from equations (112) to (118) are given in table III.

For M_0 only slightly greater than unity, the following series are useful

$$\frac{H_1}{H_0} = 1 - \frac{2\gamma}{3(\gamma+1)}\ (M_0{}^2-1)^3 + \frac{2\gamma^2}{(\gamma+1)}\ (M_0{}^2-1)^4 - \ .\ .\ .$$

$$= 1 - \frac{35}{216}\ (M_0{}^2-1)^3 + \frac{245}{864}\ (M_0{}^2-1)^4\ -\ .\ .\ . \tag{121}$$

$$\frac{\Delta S}{R} = \frac{2\gamma}{3(\gamma+1)^2}(M_o^2-1)^3 - \frac{2\gamma^2}{(\gamma+1)^3}(M_o^2-1)^4 + \ldots$$

$$= \frac{35}{216}(M_o^2-1)^3 - \frac{245}{864}(M_o^2-1)^4 + \ldots \tag{122}$$

Given $\xi \equiv \frac{p_1}{p_o}$

$(\xi > 1.0)$

$$M_o^2 = \frac{(\gamma+1)\ \xi + (\gamma-1)}{2\gamma} = \frac{6\ \xi + 1}{7} \tag{123}$$

$$M_1^2 = \frac{(\gamma-1)\ \xi + (\gamma+1)}{2\ \gamma\ \xi} = \frac{\xi + 6}{7\xi} \tag{124}$$

$$\frac{\rho_1}{\rho_o} = \frac{u_o}{u_1} = \frac{(\gamma+1)\ \xi + (\gamma-1)}{(\gamma-1)\ \xi + (\gamma+1)} = \frac{6\ \xi + 1}{\xi + 6} \tag{125}$$

$$\frac{T_1}{T_o} = \frac{a_1^2}{a_o^2} = \frac{\xi[(\gamma-1)\ \xi + (\gamma+1)]}{(\gamma+1)\xi + (\gamma-1)} = \frac{\xi(\xi + 6)}{6\ \xi + 1} \tag{126}$$

$$\frac{p_o}{H_o} = \left[\frac{(\gamma+1)[(\gamma-1)\ \xi + (\gamma+1)]}{4\gamma}\right]^{-\frac{\gamma}{\gamma-1}} = \left[\frac{6(\xi + 6)}{35}\right]^{-\frac{7}{2}} = \frac{p_1}{H_o}\left(\frac{1}{\xi}\right) \tag{127}$$

$$\frac{p_1}{H_1} = \left[\frac{(\gamma+1)[(\gamma+1)\ \xi + (\gamma-1)]}{4\gamma\ \xi}\right]^{-\frac{\gamma}{\gamma-1}} = \left[\frac{6(6\ \xi + 1)}{35\xi}\right]^{-\frac{7}{2}} \tag{128}$$

$$\frac{H_1}{H_o} = \frac{\rho_2}{\rho_a} = e^{-\frac{\Delta S}{R}} = \left[\frac{(\gamma+1)\ \xi + (\gamma-1)}{(\gamma-1)\ \xi + (\gamma+1)}\right]^{\frac{\gamma}{\gamma-1}} \xi^{-\frac{1}{\gamma-1}} = \left(\frac{6\ \xi + 1}{\xi + 6}\right)^{-\frac{7}{2}} \xi^{-\frac{5}{2}} \tag{129}$$

$$\frac{H_1}{p_0} = \left[\frac{(\gamma+1)[(\gamma+1)\,\xi + (\gamma-1)]}{4\gamma}\right]^{\frac{\gamma}{\gamma-1}} \xi^{-\frac{1}{\gamma-1}} = \left[\frac{6(6\,\xi + 1)}{35}\right]^{\frac{7}{2}} \xi^{-\frac{5}{2}} \tag{130}$$

$$\frac{\Delta S}{c_v} = (\gamma-1)\,\frac{\Delta S}{R} = -(\gamma-1)\,ln\left(\frac{H_1}{H_0}\right) = ln\,\xi - \gamma\,ln\left[\frac{(\gamma+1)\,\xi + (\gamma-1)}{(\gamma-1)\,\xi + (\gamma+1)}\right] \tag{131}$$

For weak shock waves (ξ only slightly greater than unity)

$$\frac{H_1}{H_0} = 1 - \frac{\gamma+1}{12\,\gamma^2}\,(\xi - 1)^3 + \frac{\gamma+1}{8\,\gamma^2}\,(\xi - 1)^4 - \ldots$$

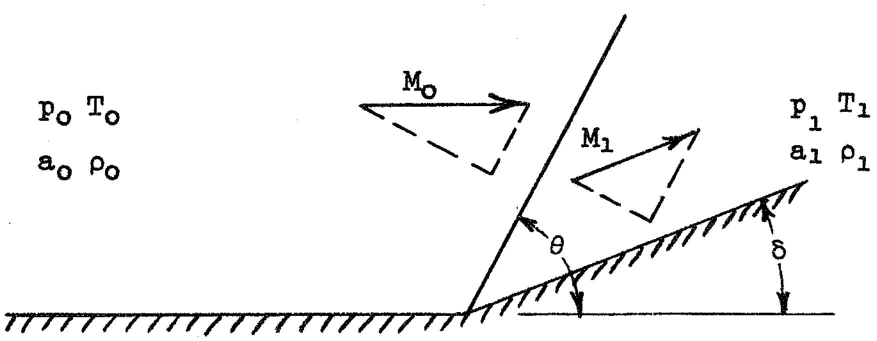
$$= 1 - \frac{5}{49}\,(\xi - 1)^3 + \frac{15}{98}\,(\xi - 1)^4 - \ldots \tag{132}$$

$$\frac{\Delta S}{R} = \frac{\gamma+1}{12\,\gamma^2}\,(\xi - 1)^3 - \frac{\gamma+1}{8\,\gamma^2}\,(\xi - 1)^4 + \ldots$$

$$= \frac{5}{49}\,(\xi - 1)^3 - \frac{15}{98}\,(\xi - 1)^4 + \ldots \tag{133}$$

B — Oblique Shock Waves

An oblique shock wave acts as a normal shock to the component of velocity directed perpendicularly against it, while the tangential component is unchanged. Hence substitution of $M_0 \sin\theta$ for M_0 and $M_1 \sin(\theta - \delta)$ for M_1 in the previous relations



provides equations (134) to (140) below. The others are determined by the condition that the flow must be deflected through an angle δ.

Again from the energy equation (the subscript 2 refers to stagnation conditions behind the shock)

$$T_2 = T_a$$

$$a_2 = a_a$$

<u>Given M_o and θ</u>

$$\frac{p_1}{p_o} \equiv \xi = \frac{2\gamma\, M_o{}^2 \sin^2\theta - (\gamma-1)}{(\gamma+1)} = \frac{7M_o{}^2 \sin^2\theta - 1}{6} \tag{134}$$

$$\frac{\rho_1}{\rho_o} = \frac{(\gamma+1)\, M_o{}^2 \sin^2\theta}{(\gamma-1)M_o{}^2 \sin^2\theta + 2} = \frac{6M_o{}^2 \sin^2\theta}{M_o{}^2 \sin^2\theta + 5} \tag{135}$$

$$\frac{T_1}{T_o} = \frac{a_1{}^2}{a_o{}^2} = \frac{[2\gamma M_o{}^2 \sin^2\theta - (\gamma-1)][(\gamma-1)M_o{}^2 \sin^2\theta + 2]}{(\gamma+1)^2 M_o{}^2 \sin^2\theta}$$

$$= \frac{(7M_o{}^2 \sin^2\theta - 1)(M_o{}^2 \sin^2\theta + 5)}{36\, M_o{}^2 \sin^2\theta} \tag{136}$$

$$M_1{}^2 \sin^2(\theta-\delta) = \frac{(\gamma-1)M_o{}^2 \sin^2\theta + 2}{(2\gamma M_o{}^2 \sin^2\theta - (\gamma-1)} = \frac{M_o{}^2 \sin^2\theta + 5}{7M_o{}^2 \sin^2\theta - 1} \tag{137}$$

$$M_1{}^2 = \frac{(\gamma+1)^2 M_o{}^4 \sin^2\theta - 4\,(M_o{}^2 \sin^2\theta - 1)\,(\gamma M_o{}^2 \sin^2\theta + 1)}{[2\gamma M_o{}^2 \sin^2\theta - (\gamma-1)]\,[(\gamma-1)M_o{}^2 \sin^2\theta + 2]}$$

$$= \frac{36\, M_o{}^4 \sin^2\theta - 5\,(M_o{}^2 \sin^2\theta - 1)(7\, M_o{}^2 \sin^2\theta + 5)}{(7\, M_o{}^2 \sin^2\theta - 1)\,(M_o{}^2 \sin^2\theta + 5)} \tag{138}$$

$$\tan \delta = \frac{2(M_0^2 \sin\theta \cos\theta - \cot\theta)}{2 + M_0^2(\gamma+1-2\sin^2\theta)} = \frac{M_0^2 \sin 2\theta - 2\cot\theta}{2 + M_0^2(\gamma + \cos 2\theta)} \tag{139}$$

$$\frac{p_1}{H_0} = \frac{2\gamma M_0^2 \sin^2\theta-(\gamma-1)}{(\gamma+1)}\left[\frac{(\gamma-1)M_0^2 + 2}{2}\right]^{-\frac{\gamma}{\gamma-1}}$$

$$= \frac{7M_0^2 \sin^2 \theta-1}{6}\left(\frac{M_0^2+5}{5}\right)^{-\frac{7}{2}} \tag{140}$$

$$\frac{p_1}{H_1} = \left[\frac{(\gamma+1)^2 M_0^2 \sin^2\theta[(\gamma-1)M_0^2+2]}{2[2\gamma M_0^2 \sin^2\theta-(\gamma-1)][(\gamma-1)M_0^2\sin^2\theta+2]}\right]^{-\frac{\gamma}{\gamma-1}}$$

$$=\left[\frac{36M_0^2 \sin^2\theta(M_0^2+5)}{5(7M_0^2\sin^2\theta-1)(M_0^2\sin^2\theta+5)}\right]^{-\frac{7}{2}} \tag{141}$$

$$\frac{H_1}{H_0} = \frac{\rho_2}{\rho_a} = e^{-\frac{\Delta S}{R}} = \left[\frac{(\gamma+1)M_0^2\sin^2\theta}{(\gamma-1)M_0^2\sin^2\theta+2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2\gamma M_0^2\sin^2\theta-(\gamma-1)}{\gamma+1}\right]^{-\frac{1}{\gamma-1}}$$

$$=\left[\frac{6M_0^2\sin^2\theta}{M_0^2\sin^2\theta+5}\right]^{\frac{7}{2}}\left[\frac{7M_0^2\sin^2\theta-1}{6}\right]^{-\frac{5}{2}} \tag{142}$$

$$\frac{H_1}{p_0} = \left[\frac{2\gamma M_0^2\sin^2\theta-(\gamma-1)}{\gamma+1}\right]^{-\frac{1}{\gamma-1}}\left[\frac{(\gamma+1)M_0^2\sin^2\theta[(\gamma-1)M_0^2+2]}{2[(\gamma-1)M_0^2\sin^2\theta+2]}\right]^{\frac{\gamma}{\gamma-1}}$$

$$=\left[\frac{7M_0^2\sin^2\theta-1}{6}\right]^{-\frac{5}{2}}\left[\frac{6M_0^2\sin^2\theta(M_0^2+5)}{5(M_0^2\sin^2\theta+5)}\right]^{\frac{7}{2}} \tag{143}$$

$$\frac{\Delta S}{c_v} = (\gamma-1)\frac{\Delta S}{R} = -(\gamma-1)\ ln\left(\frac{H_1}{H_0}\right)$$

$$= ln\left[\frac{2\gamma M_0^2\sin^2\theta-(\gamma-1)}{\gamma+1}\right] - \gamma\ ln\left[\frac{(\gamma+1)M_0^2\sin^2\theta}{(\gamma-1)M_0^2\sin^2\theta+2}\right] \qquad (144)$$

$$\frac{V_1^2}{V_0^2} = \frac{u_1+v_1^2}{u_0^2} = \frac{(\gamma+1)^2M_0^4\ \sin^2\theta-4(M_0^2\sin^2\theta-1)(\gamma M_0^2\sin^2\theta+1)}{(\gamma+1)^2M_0^4\sin^2\theta}$$

$$= \frac{36\ M_0^4\ \sin^2\theta-5(M_0^2\sin^2\theta-1)(7\ M_0^2\sin^2\theta+5)}{36\ M_0^4\ \sin^2\theta} \qquad (145)$$

For weak shock waves ($M_0 \sin\theta$ only slightly greater than unity) these series are useful

$$\frac{H_1}{H_0} = 1 - \frac{2\gamma}{3(\gamma+1)^2}(M_0^2\sin^2\theta-1)^3 + \frac{2\gamma^2}{(\gamma+1)^3}(M_0^2\sin^2\theta-1)^4 - \ldots$$

$$= 1 - \frac{35}{216}(M_0^2\sin^2\theta-1)^3 + \frac{245}{864}(M_0^2\sin^2\theta-1)^4 - \ldots \qquad (146)$$

$$\frac{\Delta S}{R} = \frac{2\gamma}{3(\gamma+1)^2}(M_0^2\sin^2\theta-1)^3 - \frac{2\gamma^2}{(\gamma+1)^3}(M_0^2\sin^2\theta-1)^4 + \ldots$$

$$= \frac{35}{216}(M_0^2\sin^2\theta-1)^3 - \frac{245}{864}(M_0^2\sin^2\theta-1)^4 + \ldots \qquad (147)$$

Given θ and δ

$$M_o^2 = \frac{2(\cot\theta + \tan\delta)}{\sin 2\theta - \tan\delta\,(\gamma + \cos 2\theta)} \tag{148}$$

Given M_o and δ

No explicit relations can be obtained. The following series (which is identical to that for isentropic flow up to and including the term in δ^2) is used in Busemann's airfoil theory (reference 3) for small values of δ (in radians):

$$\frac{p_1}{p_o} \equiv \xi = 1 + \frac{\gamma M_o^2}{\sqrt{M_o^2-1}}\,\delta + \frac{\gamma(\gamma+1)M_o^6 - 4\gamma M_o^2(M_o^2-1)}{4(M_o^2-1)^2}\,\delta^2 + \ldots \tag{149}$$

Given $\xi \equiv \frac{p_1}{p_o}$

$$M_o^2 \sin^2\theta = \frac{(\gamma+1)\,\xi + (\gamma-1)}{2\gamma} = \frac{6\,\xi + 1}{7} \tag{150}$$

$$M_1^2 \sin^2(\theta-\delta) = \frac{(\gamma-1)\,\xi + (\gamma+1)}{2\gamma\xi} = \frac{\xi + 6}{7\xi} \tag{151}$$

$$M_1^2 = \frac{M_o^2[(\gamma+1)\,\xi + (\gamma-1)] - 2(\xi^2-1)}{\xi[(\gamma-1)\,\xi + (\gamma+1)]} = \frac{M_o^2(6\,\xi + 1) - 5(\xi^2-1)}{\xi\,(\xi + 6)} \tag{152}$$

$$\frac{\rho_1}{\rho_o} = \frac{(\gamma+1)\,\xi + (\gamma-1)}{(\gamma-1)\,\xi + (\gamma+1)} = \frac{6\xi + 1}{\xi + 6} \tag{153}$$

$$\frac{T_1}{T_0} = \frac{a_1^2}{a_0^2} = \frac{\xi\,[(\gamma-1)\,\xi + (\gamma+1)]}{(\gamma+1)\xi + (\gamma-1)} = \frac{\xi\,(\xi + 6)}{6\xi + 1} \tag{154}$$

$$\tan^2 \delta = \left(\frac{\xi - 1}{\gamma M_0^2 - \xi + 1}\right)^2 \frac{2\gamma M_0^2 - (\gamma-1) - (\gamma+1)\xi}{(\gamma+1)\,\xi + (\gamma-1)}$$

$$= \left[\frac{5(\xi - 1)}{7M_0^2 - 5(\xi - 1)}\right]^2 \frac{7M_0^2 - (6\xi + 1)}{6\xi + 1} \tag{155}$$

$$\frac{H_1}{H_0} = \frac{\rho_2}{\rho_a} = e^{-\frac{\Delta S}{R}} = \left[\frac{(\gamma+1)\,\xi + (\gamma-1)}{(\gamma-1)\,\xi + (\gamma+1)}\right]^{\frac{\gamma}{\gamma-1}} \xi^{-\frac{1}{\gamma-1}} = \left[\frac{6\xi + 1}{\xi + 6}\right]^{\frac{7}{2}} \xi^{-\frac{5}{2}} \tag{156}$$

$$\frac{V_1^2}{V_0^2} = \frac{u_1^2 + v_1^2}{u_0^2} = 1 - \frac{2(\xi^2 - 1)}{M_0^2[(\gamma+1)\,\xi + (\gamma-1)]} = 1 - \frac{5(\xi^2 - 1)}{M_0^2(6\xi + 1)} \tag{157}$$

Equations (132) and (133) of part A of this section are applicable to oblique shocks as well as to normal shocks.

Use of Tables

The following values for oblique shock waves may be read from table III provided $M_0 \sin \theta$ is used instead of M_0 in the first column.

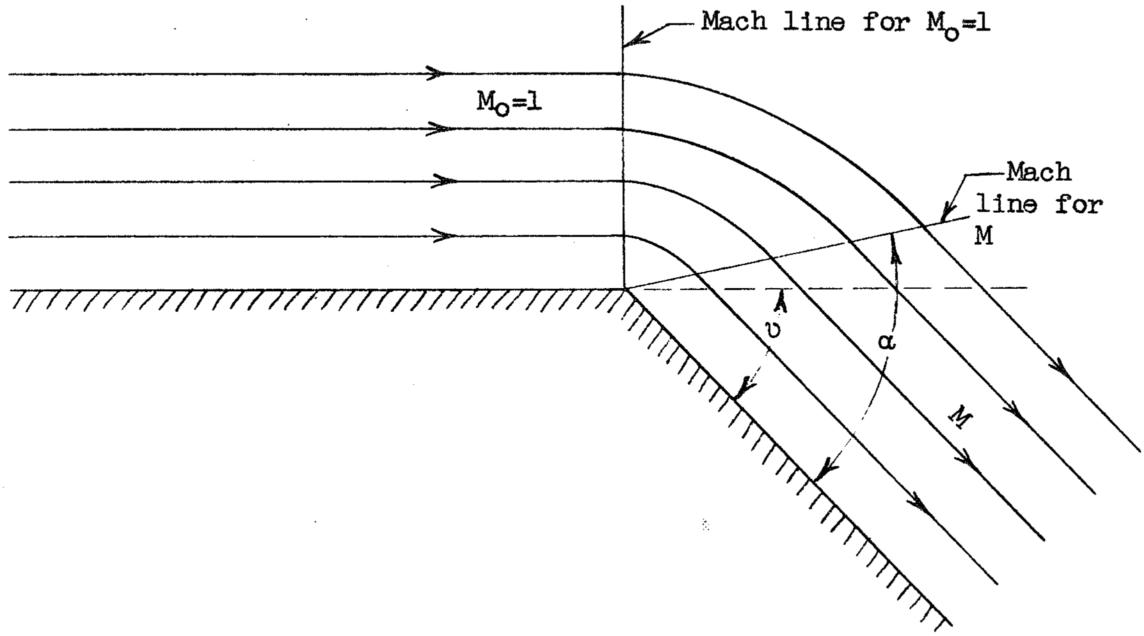
$$\frac{p_1}{p_0},\quad \frac{\rho_1}{\rho_0},\quad \frac{T_1}{T_0},\quad \frac{a_1}{a_0},\quad \frac{H_1}{H_0}$$

Furthermore, since flow through weak shock waves is nearly isentropic, compressions through small angles δ may be computed from table II as if they were expansions reversed.

IV – EXPANSION AROUND A CORNER

A – Prandtl-Meyer Expansion

The following equations are valid for two-dimensional, isentropic, irrotational flow of a perfect gas.



The final equation for the angle through which a stream must turn to expand from $M_o = 1$ to a Mach number M is (υ in degrees):

$$\upsilon = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2-1)} - (90^\circ - \alpha)$$

or

$$\upsilon = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2-1)} - \left(90^\circ - \sin^{-1} \frac{1}{M}\right) \qquad (158)$$

The pressure ratio $\frac{p}{H}$ corresponding to the Mach number M is given by (υ and α in degrees):

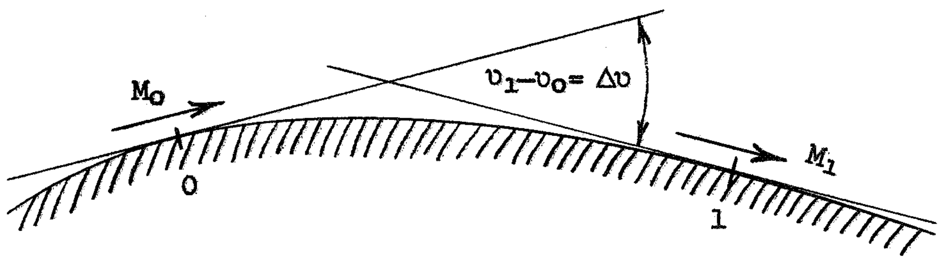
$$\left(\frac{p}{H}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{\gamma+1}\left\{1 + \cos 2\sqrt{\frac{\gamma-1}{\gamma+1}}\,[\upsilon + (90^{\circ} - \alpha)]\right\}$$

or

$$\left(\frac{p}{H}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{\gamma+1}\left\{1 + \cos 2\sqrt{\frac{\gamma-1}{\gamma+1}}\left[\upsilon + \left(90^{\circ} - \sin^{-1}\frac{1}{M}\right)\right]\right\} \qquad (159)$$

Numerical values for υ, α, and p/H will be found in table II as a function of M.

The above equations and the numerical values in table II apply also to the flow along a convex curved surface in the absence of external or reflected disturbances in the region.



Use of Table II

Consider stations 0 and 1. If M_0 and M_1, or p_0/H_0 and p_1/H_0, or any other conditions at stations 0 and 1 are known, the angle through which the stream must turn to expand from M_0 to M_1 may be found by referring to υ_1 and υ_0 in table II and taking the difference $\Delta\upsilon$.

If M_0 and $\Delta\upsilon$ are known, the conditions at station 1 may be found by obtaining $\upsilon_1 = \upsilon_0 + \Delta\upsilon$ and looking in table II under this value of υ_1.

For expansions through small angles, p_1/p_0 may be expressed in series where $\Delta\upsilon = \upsilon_1 - \upsilon_0$ (in radians) as follows:

$$\frac{p_1}{p_0} = 1 + \frac{\gamma M_0^2}{\sqrt{M_0^2-1}}\Delta\upsilon + \frac{\gamma(\gamma+1)M_0^6-4\gamma M_0^2(M_0^2-1)}{4(M_0^2-1)^2}(\Delta\upsilon)^2$$

$$+ \frac{\gamma}{2(M_0^2-1)^{\frac{7}{2}}}\left\{\frac{\gamma+1}{4}M_0^{10} - \frac{(-2\gamma^2+7\gamma+5)}{4}M_0^8\right.$$

$$+\left[\frac{(-2\gamma^2+7\gamma+5)^2}{16(\gamma+1)} + \frac{(-4\gamma^4+28\gamma^3+11\gamma^2-8\gamma-3)}{24(\gamma+1)} + \frac{3}{4}\right]M_0^6$$

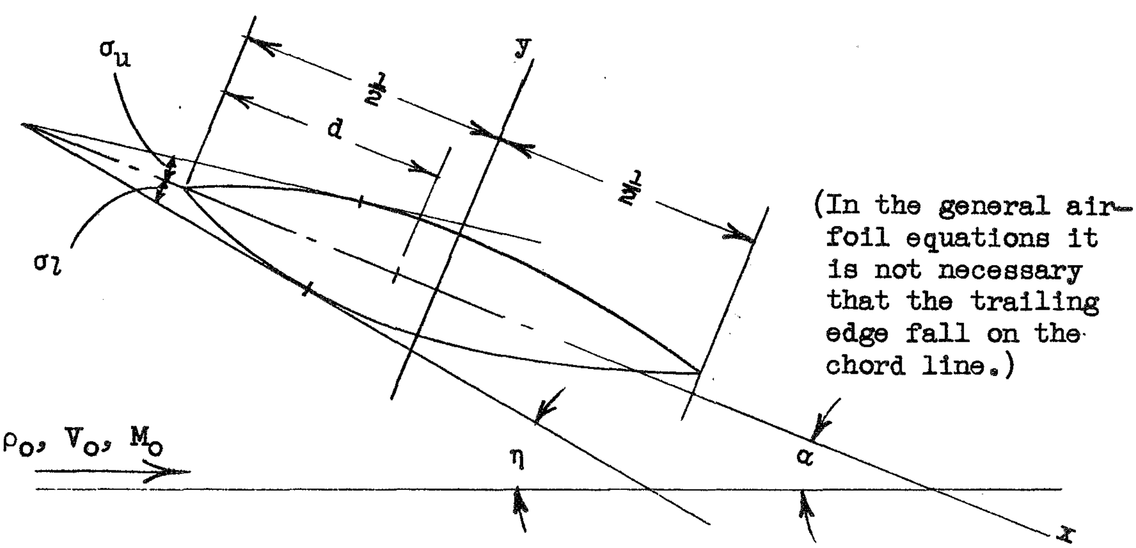
$$\left. - 2M_0^4 + \frac{4}{3}M_0^2\right\}(\Delta\upsilon)^3 + \ldots\ldots \qquad (160)$$

Up to and including the term in $(\Delta\upsilon)$ this series is identical to that for the pressure ratio across oblique shocks. (See equation (149).)

V – AIRFOIL THEORY

A – Small-Perturbation Section Theory

It is assumed that the angle of inclination η of the airfoil surface relative to the free-stream direction is everywhere small. This implies that the angle of attack α is small, that the surface of the airfoil makes at all points a small angle σ



with the chord line, and that the leading and trailing edges are sharp. The theory is not valid below the free-stream Mach number at which the flow at any point in the field becomes subsonic.

The pressure at any point on the airfoil is given by

$$\frac{p-p_o}{\frac{1}{2}\rho_o V_o^2} = C_1\eta + C_2\eta^2 + \text{(terms of higher order)} \tag{161}$$

where C_1 and C_2 are functions of M_o given by

$$C_1 = \frac{2}{\sqrt{M_o^2-1}} \tag{162}$$

$$C_2 = \frac{(\gamma+1)M_o^4-4(M_o^2-1)}{2(M_o^2-1)^2} \tag{163}$$

Values of C_1 and C_2 are listed in table IV.

The equations of part A that follow are based on the work of C. N. H. Lock (reference 4) and have been deduced from Busemann's approximation (reference 3), which retains the first two terms in equation (161). Corresponding equations based on Ackeret's approximation (reference 5), which retains only the first term, can be obtained by setting $C_2=0$ in the given equations. The error resulting from the approximations used in Busemann's analysis is of the order of η^3; whereas in Ackeret's analysis it is of the order of η^2.

To evaluate the aerodynamic coefficients for any given airfoil section the following integrals are required:

$$
\left.
\begin{aligned}
I_0 &= \int \sigma \, dx \\
I_1 &= \int \sigma^2 \, dx \\
I_2 &= \int \sigma^3 \, dx \\
I_3 &= \int \sigma x \, dx \\
I_4 &= \int \sigma^2 x \, dx
\end{aligned}
\right\} \tag{164}
$$

In general, all integrals are evaluated over both surfaces of the airfoil from $x = -1/2$ to $x = +1/2$. The value of σ at any point on either surface is positive when, for an observer moving from the leading edge to the trailing edge, the absolute value of the ordinate of that surface is increasing (see foregoing diagram). All angles are in radian measure. All linear dimensions are referred to the airfoil chord.

For any airfoil with both the leading and trailing edges on the chord line, $I_0 \equiv 0$ for each surface.

<u>General airfoil</u>.– The force coefficients for any general airfoil are

$$c_l = 2C_1\alpha + C_1(I_{0_l} - I_{0_u}) + C_2(I_{1_l} - I_{1_u}) \tag{165}$$

$$
\begin{aligned}
c_d = {} & 2C_1\alpha^2 + 2C_1(I_{0_l} - I_{0_u})\alpha + C_1(I_{1_l} + I_{1_u}) \\
& + 3C_2(I_{1_l} - I_{1_u})\alpha + C_2(I_{2_l} + I_{2_u})
\end{aligned} \tag{166}
$$

$$
\begin{aligned}
c_{m_{\frac{1}{2}}} = {} & -C_1(I_{3_l} - I_{3_u}) - 2C_2(I_{3_l} + I_{3_u})\alpha \\
& - C_2(I_{4_l} - I_{4_u})
\end{aligned} \tag{167}
$$

$$c_{m_{\frac{d}{c}}} = c_{m_{\frac{1}{2}}} + \left(\frac{d}{c} - \frac{1}{2}\right) c_l \tag{168}$$

It should be noted that the formulae for c_d given in this section are for the total wave drag only, and do not include any effects of viscosity.

For any airfoil, $(I_{3_l} + I_{3_u})$ is equal to minus the cross-sectional area of the airfoil.

Symmetrical airfoils.- Equations for calculating the aerodynamic coefficients for five types are listed as follows:

1. Symmetry about the chord line (x-axis).- In this case $\sigma_l = \sigma_u$ and the general equations reduce to

$$c_l = 2C_1\alpha \tag{169}$$

$$c_d = 2C_1\alpha^2 + 2C_1I_{1_l} + 2C_2I_{2_l} \tag{170}$$

$$c_{m_{\frac{1}{2}}} = -4C_2I_{3_l}\alpha \tag{171}$$

2. Symmetry about the normal to the chord line at midchord (y-axis).- In this case $\sigma(x) = -\sigma(-x)$ and the general equations reduce to

$$c_l = 2C_1\alpha + C_2(I_1 - I_{1_l} - I_{1_u}) \tag{172}$$

$$c_d = 2C_1\alpha^2 + C_1(I_{1_l} + I_{1_u}) + 3C_2(I_{1_l} - I_{1_u})\alpha \tag{173}$$

$$c_{m_{\frac{1}{2}}} = -C_1(I_{3_l} - I_{3_u}) - 2C_2(I_{3_l} + I_{3_u})\alpha \tag{174}$$

3. Symmetry about both the chord line and the normal to the chord line at midchord (x- and y-axes).-

$$c_l = 2C_1\alpha \tag{175}$$

$$c_d = 2C_1\alpha^2 + 2C_1I_{1_l} \tag{176}$$

$$c_{m_{\frac{1}{2}}} = -4C_2I_{3_l}\alpha \tag{177}$$

4. Antisymmetry about the midchord point (origin).- In this case $\sigma_u(x) = -\sigma_l(-x)$ and the general equations reduce to

$$c_l = 2C_1\alpha + 2C_1I_{o_l} \tag{178}$$

$$c_d = 2C_1\alpha^2 + 4C_1I_{o_l}\alpha + 2C_1I_{1_l} \tag{179}$$

$$c_{m\frac{1}{2}} = -4C_2I_{3_l}\alpha - 2C_2I_{4_l} \tag{180}$$

Because of the conditions of symmetry, in cases 1, 3, and 4, the integrals (164) need be evaluated over one surface only.

Specific types of airfoils.- Equations for calculating the aerodynamic coefficients for specific types of airfoils are as follows:

1. Airfoils made up of segments of straight lines and circular arcs.- Consider a portion of an airfoil surface of circular-arc form with a radius r (in chord lengths) and a radius center on the normal to the chord line at $x = s$. For a thin airfoil, to a sufficient degree of approximation,

$$\sigma = \sigma_1 + \sigma_2 x \tag{181}$$

where for convex surfaces (both upper and lower)

$$\sigma_1 = \frac{s}{r}, \qquad \sigma_2 = -\frac{1}{r}$$

and for concave surfaces (both upper and lower)

$$\sigma_1 = -\frac{s}{r}, \qquad \sigma_2 = \frac{1}{r}$$

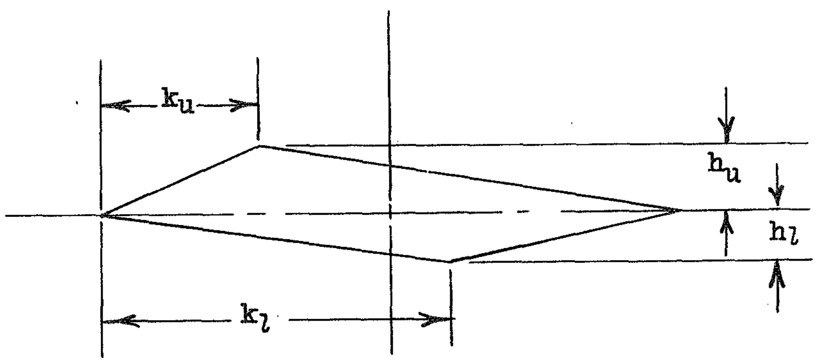
For a portion of the circular-arc surface between the limits $x = m$ and $x = n$, the contribution of this portion to the integrals (164) is

$$\left.\begin{aligned}
\Delta I_{0_{mn}} &= \sigma_1(n-m) + \frac{1}{2}\sigma_2(n^2-m^2) \\
\Delta I_{1_{mn}} &= \sigma_1^2(n-m) + \sigma_1\sigma_2(n^2-m^2) + \frac{1}{3}\sigma_2^2(n^3-m^3) \\
\Delta I_{2_{mn}} &= \sigma_1^3(n-m) + \frac{3}{2}\sigma_1^2\sigma_2(n^2-m^2) + \sigma_1\sigma_2^2(n^3-m^3) \\
&\quad + \frac{1}{4}\sigma_2^3(n^4-m^4) \\
\Delta I_{3_{mn}} &= \frac{1}{2}\sigma_1(n^2-m^2) + \frac{1}{3}\sigma_2(n^3-m^3) \\
\Delta I_{4_{mn}} &= \frac{1}{2}\sigma_1^2(n^2-m^2) + \frac{2}{3}\sigma_1\sigma_2(n^3-m^3) + \frac{1}{4}\sigma_2\ (n^4-m^4)
\end{aligned}\right\} \tag{182}$$

(Note that m, n, and s are measured from the midchord point)

The corresponding expressions for a straight-line element of surface are obtained by putting $\sigma_2 = 0$ and replacing σ_1 with the angle of inclination of that element relative to the chord line with due regard to signs as defined immediately after equations (164).

2. Double-wedge airfoil.-



For the general double-wedge airfoil as shown

$$c_l = 2C_1\alpha + C_2\left[h_l^2F_1(k_l) - h_u^2F_1(k_u)\right] \tag{183}$$

$$\begin{aligned} c_d = {} & 2C_1\alpha^2 + C_1\left[h_l^2F_1(k_l) + h_u^2F_1(k_u)\right] \\ & + 3C_2\left[h_l^2F_1(k_l) - h_u^2F_1(k_u)\right]\alpha \\ & + C_2\left[h_l^3F_2(k_l) + h_u^3F_2(k_u)\right] \end{aligned} \tag{184}$$

$$\begin{aligned} c_{m_{\frac{1}{2}}} = {} & \frac{1}{2}C_1(h_l-h_u) + C_2(h_l+h_u)\alpha \\ & - \frac{1}{2}C_2\left[h_l^2F_4(k_l) - h_u^2F_4(k_u)\right] \end{aligned} \tag{185}$$

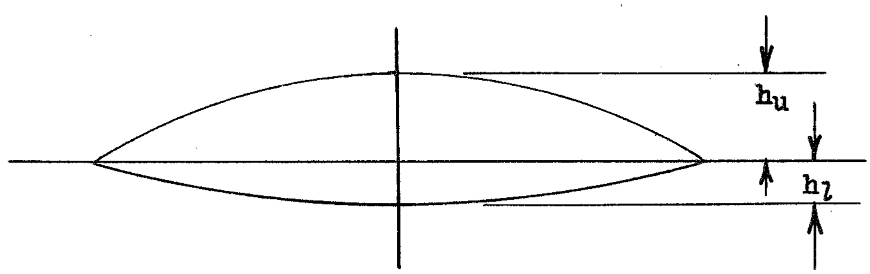
where the functions $F(k)$ are defined by

$$\left.\begin{aligned} F_1(k) &= \frac{1}{k} - \frac{1}{k-1} \\ F_2(k) &= \frac{1}{k^2} - \frac{1}{(k-1)^2} \\ F_4(k) &= \frac{k-1}{k} - \frac{k}{k-1} \end{aligned}\right\} \tag{186}$$

Values of these functions are given in the following table:

k	F_1	F_2	F_4	k	F_1	F_2	F_4
0	– –	– –	– –	.55	4.040	–1.632	0.404
.05	21.053	398.892	–18.947	.60	4.167	–3.472	0.833
.10	11.111	98.765	– 8.889	.65	4.395	–5.796	1.319
.15	7.843	43.060	– 5.491	.70	4.762	–9.070	1.904
.20	6.250	23.437	– 3.750	.75	5.333	–14.222	2.667
.25	5.333	14.222	– 2.667	.80	6.250	–23.437	3.750
.30	4.762	9.070	– 1.904	.85	7.843	–43.060	5.491
.35	4.395	5.796	– 1.319	.90	11.111	–98.765	8.889
.40	4.167	3.472	– 0.833	.95	21.053	–398.892	18.947
.45	4.040	1.632	– 0.404	1.00	– –	– –	– –
.50	4.000	0	0				

3. Biconvex airfoil.–



For the general biconvex airfoil made up of two circular arcs as shown

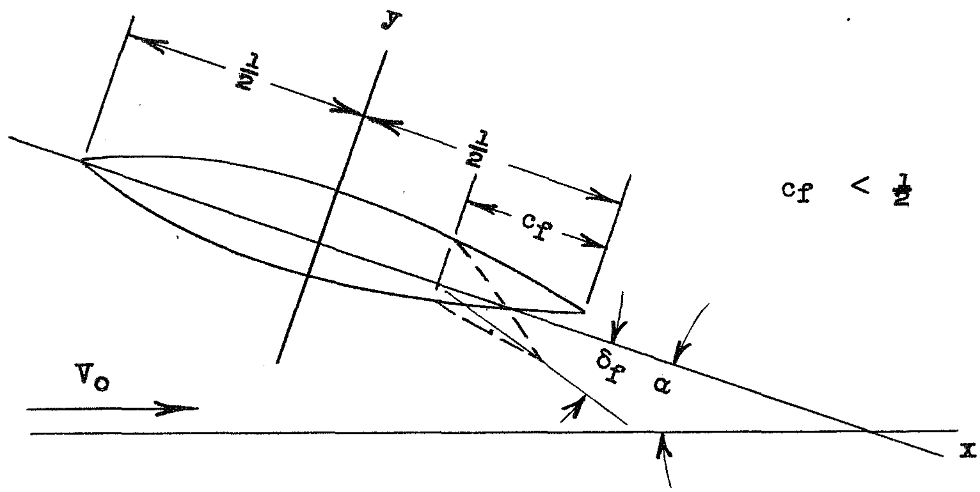
$$c_l = 2C_1\alpha + \frac{16}{3}C_2(h_l^2-h_u^2) \tag{187}$$

$$c_d = 2C_1\alpha^2 + \frac{16}{3}C_1(h_l^2+h_u^2) + 16C_2(h_l^2-h_u^2)\alpha \tag{188}$$

$$c_{m_{\frac{1}{2}}} = \frac{2}{3}C_1(h_l-h_u) + \frac{4}{3}C_2(h_l+h_u)\alpha \tag{189}$$

<u>Airfoil with Flap</u>.— Equations for calculating the aerodynamic coefficients of an airfoil with flap are as follows:

1. <u>General airfoil with flap</u>.—



For an airfoil with a plain flap of chord $c_f < \frac{1}{2}$ as shown, the increments to the aerodynamic coefficients as a result of the deflection of the flap are

$$(\Delta c_l)_f = 2C_1 c_f \delta_f + 2C_2 (I_{o_l} + I_{o_u})_f \, \delta_f \tag{190}$$

$$(\Delta c_d)_f = 2C_1 c_f (2\alpha + \delta_f)\delta_f + 2C_1 (I_{o_l} - I_{o_u}) \, \delta_f$$

$$+ 3C_2 (I_{o_l} + I_{o_u})_f (2\alpha + \delta_f)\delta_f + 3C_2 (I_{1_l} - I_{1_u})_f \, \delta_f \tag{191}$$

$$\left(\Delta c_{m_{\frac{1}{2}}}\right)_f = - C_1 c_f (1 - c_f)\delta_f - 2C_2 (I_{3_l} + I_{3_u})\delta_f \tag{192}$$

where the subscript f to the integral expressions indicates that the integrals (equation 164) are evaluated in this case over the chord of the flap, that is, from $x = \frac{1}{2} - c_f$ to $x = \frac{1}{2}$.

The hinge-moment coefficient for the flap is

$$c_h = - C_1(\alpha+\delta_f) + \frac{C_1}{c_f^2}\left(\frac{1}{2} - c_f\right)(I_{0_l}-I_{0_u})_f - \frac{C_1}{c_f^2}(I_{3l}-I_{3u})_f$$

$$+ 2\frac{C_2}{c_f^2}\left(\frac{1}{2} - c_f\right)(I_{0_l}+I_{0_u})_f(\alpha+\delta_f) - 2\frac{C_2}{c_f^2}(I_{3_l}+I_{3_u})_f(\alpha+\delta_f)$$

$$+ \frac{C_2}{c_f^2}\left(\frac{1}{2} - c_f\right)(I_{1_l}-I_{1_u})_f - \frac{C_2}{c_f^2}(I_{4l}-I_{4u})_f \tag{193}$$

2. Airfoil with straight-sided symmetrical flap.- For any airfoil with a symmetrical flap having straight sides each of which make an angle τ with the chord line of the airfoil at the trailing edge,

$$(\Delta c_l)_f = 2(C_1-2C_2\tau)\ c_f\delta_f \tag{194}$$

$$(\Delta c_d)_f = 2(C_1- C_2\tau)\ c_f\ (2\alpha+\delta_f)\delta_f \tag{195}$$

$$\left(\Delta c_{m_{\frac{1}{2}}}\right)_f = - (C_1-2C_2\tau)\ c_f\ (1-c_f)\delta_f \tag{196}$$

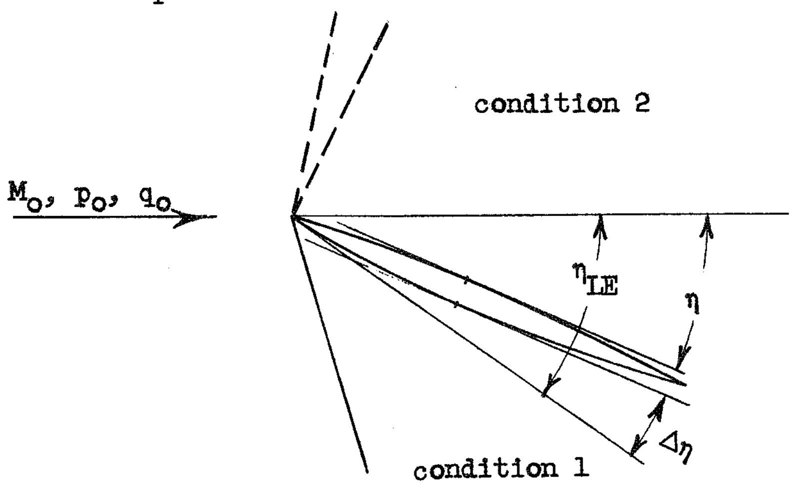
$$c_h = - (C_1-2C_2\tau)\ (\alpha+\delta_f) \tag{197}$$

Limits of the Theory.— All of the foregoing equations of section V-A are theoretically valid only if the flow field is everywhere supersonic. For any given case the minimum free-stream Mach number at which this condition exists can be determined from the data of section VI as follows:

For the angle of attack in question, determine the maximum angle θ_s through which the stream is deflected in a compressive direction at the leading edge of the airfoil. The minimum free-stream Mach number above which the foregoing equations are valid is then given by the curve in figure 2 which defines a Mach number of unity aft of the shock wave for a wedge.

B – Large-Deflection Section Theory

With the assumption of zero viscosity, the pressure distribution over a given airfoil at a given angle can be plotted with good accuracy with the aid of the data on flow about wedges from section VI and on expansion around a corner from section IV. The section force and moment coefficients can then be found more accurately than with the small-perturbation theory of part A by graphical or numerical integration of the pressure distribution.



Two conditions are possible with regard to the flow over the surface of an airfoil:

Condition 1 – The flow at the leading edge is characterized by a compression through a shock wave, as on the lower surface of the airfoil in the preceding figure.

Condition 2 — The flow at the leading edge is characterized by an expansion around a corner, as on the upper surface of the airfoil in the figure.

The method to be followed in plotting the pressure distribution in each case is outlined below.

Condition 1.— The pressure coefficient P_{LE} at the leading edge just behind the shock wave is given by

$$P_{LE} = \frac{p_{LE}-p_o}{q_o} = \frac{1}{\left(\frac{q_o}{p_o}\right)} \left\{\frac{p_{LE}}{p_o} - 1\right\} \tag{198}$$

The value of (p_{LE}/p_o) in this equation is found from figure 3, where, for this case, δ is replaced by η_{LE} and (p_1/p_o) by (p_{LE}/p_o). The value of (q_o/p_o) is found from table II as a function of M_o.

The pressure coefficient P at any other point on the surface is then given by

$$P = \frac{p-p_o}{q_o} = \frac{1}{\left(\frac{q_o}{p_o}\right)} \left\{\frac{p}{p_{LE}} \quad \frac{p_{LE}}{p_o} \quad - 1\right\} \tag{199}$$

The values of (q_o/p_o) and (p_{LE}/p_o) are already known.

The value of (p/p_{LE}) can be found as follows:

Determine the Mach number M_{LE} from figure 4 where δ is replaced by η_{LE} and M_1 by M_{LE}. Read the corresponding values of υ_{LE} and (p_{LE}/H) from table II as a function of M_{LE}. Compute the value of υ at the point in question from the equation

$$\upsilon = \upsilon_{LE} + \Delta\eta \tag{200}$$

where $\Delta\eta$ is the change in angle between the leading edge and the given point. Find the corresponding value of (p/H) from Table II.

The desired quantity is then given by

$$\left(\frac{p}{p_{LE}}\right) = \frac{\left(\frac{p}{H}\right)}{\left(\frac{p_{LE}}{H}\right)} \tag{201}$$

Condition 2.– In this case the pressure coefficient at any point on the surface, including the leading edge, is given by

$$P = \frac{p-p_o}{q_o} = \frac{1}{\left(\frac{q_o}{p_o}\right)} \left\{\frac{p}{p_o} - 1\right\} \tag{202}$$

The value of (q_o/p_o) is determined from table II as a function of M_o. To determine p/p_o, first find υ_o and p_o/H from table II as functions of M_o. Find υ at the point in question from

$$\upsilon = \upsilon_o + \eta \tag{203}$$

where η is the angle of inclination of the surface with respect to the free stream, and determine the corresponding value of (p/H) from table II. The desired ratio is then given by

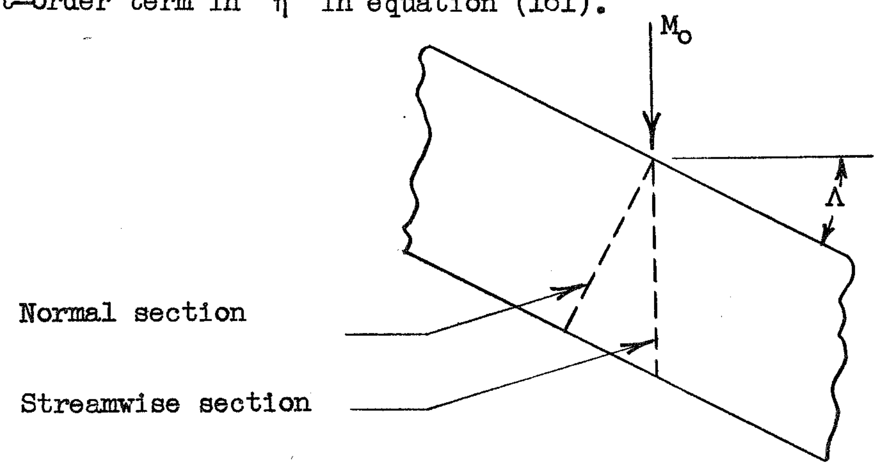
$$\left(\frac{p}{p_o}\right) = \frac{\left(\frac{p}{H}\right)}{\left(\frac{p_o}{H}\right)} \tag{204}$$

Limits of the theory.– The limit of the large-deflection theory as regards minimum Mach number is the same as that outlined for the small-perturbation theory, i.e., the flow field must everywhere be supersonic in order that the theory be applicable.

It is to be noted that the application of Prandtl–Meyer flow to the expansion immediately behind a shock wave (as is done in the analysis of condition 1,) is not strictly justified, even in a nonviscous fluid. However, it is a good approximation to actual conditions.

C. Small–Perturbation Sweepback Theory

The effect of sweepback for a constant–chord wing of infinite span has been determined by Busemann (reference 6) and by Ludwieg and Weber (reference 7). The theory is based on the same assumptions as the small–perturbation theory of part A. In applying the theory, the free–stream Mach number is resolved into a component parallel to the leading edge $M_o \sin \Lambda$, which has no effect on the surface pressures, and a component perpendicular to the leading edge $M_o \cos \Lambda$. The lift force acting perpendicular to the free–stream direction, and the drag force acting in the horizontal plane perpendicular to the leading edge are then the same as the lift and drag forces that would act on an unswept wing at a stream Mach number of $M_o \cos \Lambda$. The resulting equations correspond to Ackeret's equations for airfoil sections; that is, they are based on the retention of only the first–order term in η in equation (161).



Two cases are considered with respect to the angle of attack:

(1) For the case in which the wing is rotated about an axis parallel to the leading and trailing edges, the force coefficients are

$$c_l = \frac{4 \cos \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} \alpha + \frac{2 \cos^2 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} \left(I_{o_l} - I_{o_u}\right) \qquad (205)$$

$$c_d = \frac{4 \cos \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} \alpha^2 + \frac{4 \cos^2 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} (I_{o_l} - I_{o_u}) \alpha$$

$$+ \frac{2 \cos^2 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} (I_{1_l} + I_{1_u}) \tag{206}$$

(2) For the case in which the wing is rotated about an axis perpendicular to the streamwise direction the following equations apply

$$c_l = \frac{4 \cos^3 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} \alpha + \frac{2 \cos^2 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} (I_{o_l} - I_{o_u}) \tag{207}$$

$$c_d = \frac{4 \cos^5 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} \alpha^2 + \frac{4 \cos^4 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} (I_{o_l} - I_{o_u}) \alpha$$

$$+ \frac{2 \cos^2 \Lambda}{\sqrt{M_o^2 \cos^2 \Lambda - 1}} (I_{1_l} - I_{1_u}) \tag{208}$$

The angle of attack α (in radians) in both cases is the angle between the undisturbed stream and the chordline of a streamwise section of the wing. The integrals I, defined by equations (164), are here evaluated for a streamwise section of the wing, that is, for a section taken parallel to the general undisturbed flow. (The integrals I_O are zero for all airfoils in which the chordline passes through both the leading and trailing edges.)

If it is desired to evaluate the integrals I for a normal section of the wing, that is, for a section taken normal to the leading edge, the foregoing equations can be modified with the following relationships

$$I_o = (I_o)_n \tag{209}$$

$$I_1 = (I_1)_n \cos \Lambda \tag{210}$$

where the subscript n refers to a section taken normal to the leading edge.

The center of pressure of the swept-back wing at an angle of attack α has the same chordwise position (in percent chord) as would a normal section of the wing set at an angle of attack of $\alpha/\cos\Lambda$ (case 1), or $\alpha \cos \Lambda$ (case 2) in a two-dimensional stream of Mach number $M_o \cos \Lambda$.

For a given angle of attack α, the foregoing equations are theoretically invalid below the free-stream Mach number M_o at which the quantity $M_o \cos \Lambda$ becomes equal to the limiting Mach number for the normal section of the wing when set at an angle $\alpha/\cos \Lambda$ (case 1), or $\alpha \cos \Lambda$ (case 2) in two-dimensional flow (part A).

VI – FLOW ABOUT WEDGES AND CONES

When a body moves through air at a uniform speed greater than that of sound, a shock wave is formed which remains fixed relative to the body. A pointed shape, that is, a two-dimensional wedge or a cone, forms an oblique shock wave the characteristics of which are determined by the vertical angle and the free-stream Mach number. At high Mach numbers the shock wave originates from the point and forms an angle with the body axis that varies inversely with the Mach number. When the speed is reduced to a certain critical value which depends upon the vertical angle, the shock wave becomes detached from the point and stands ahead of the body. A comparison of the conditions under which shock waves become detached from wedges and cones is shown in figure 5 which presents the maximum wave angle and the minimum Mach number at which shock waves are attached. A comparison of the flow conditions about both cones and wedges is shown in figure 2. These characteristics were determined by theoretical computations that agree excellently with experimental results.

A – Flow About Wedges

The equations which describe the conditions about two-dimensional wedges in supersonic flow are given in section IV, part B, Oblique Shock Waves. As explained in that section, the following values may be determined from table III if $M_o \sin\theta$ is used in place of M_o in the first column:

$$\frac{p_1}{p_o},\ \frac{\rho_1}{\rho_o},\ \frac{T_1}{T_o},\ \frac{a_1}{a_o},\ \frac{H_1}{H_o}$$

Figures 3, 4, and 6 give the characteristics (i.e., p_1/p_o, M_1, and θ_w,) of oblique shock waves for wedges of a given semivertical angle δ as functions of free-stream Mach number M_o.

B – Flow About Cones

The conditions about cones in supersonic flow have been calculated by G. I. Taylor and J. W. Maccoll, and have been reported in references 1 and 2. The results of these calculations are shown in figures 7 and 8; the former shows the wave angle θ_w and the latter the pressure coefficient P for cones of various semivertical angles θ_s as a function of Mach number M_o.

APPENDIX A

VISCOSITY OF AIR

The viscosity of air is sensibly independent of pressure, while its variation with temperature is closely represented by

$$\frac{\mu_2}{\mu_1} = \left(\frac{T_2}{T_1}\right)^{0.76} \tag{A-1}$$

or more accurately by Sutherland's formula (T in $^{o}R = {}^{o}F + 460$):

$$\frac{\mu_2}{\mu_1} = \frac{T_1 + 216}{T_2 + 216}\left(\frac{T_2}{T_1}\right)^{3/2} \tag{A-2}$$

The following values of μ are based on Sutherland's formula:

T (°F)	μ $\left(\frac{\text{lb sec}}{\text{ft}^2} \times 10^{-6}\right)$	T (°F)	μ $\left(\frac{\text{lb sec}}{\text{ft}^2} \times 10^{-6}\right)$	T (°F)	μ $\left(\frac{\text{lb sec}}{\text{ft}^2} \times 10^{-6}\right)$
-100	0.274	-30	0.319	40	0.361
- 90	.280	-20	.325	50	.366
- 80	.287	-10	.331	60	.372
- 70	.294	0	.337	70	.378
- 60	.300	10	.343	80	.383
- 50	.306	20	.349	90	.389
- 40	.313	30	.355	100	.394

APPENDIX B

REYNOLDS NUMBER

Reynolds Number is defined as

$$Re = \frac{\rho V_o l}{\mu} = \frac{V_o l}{\upsilon} \tag{B-1}$$

where l is a characteristic linear dimension.

Approximately, for airfoils at sea level,

$$Re = 10,000\ (V_o \text{ in mph})(\text{chord in feet}) \tag{B-2}$$

The variation of Reynolds number per foot with Mach number for vario altitudes is given in figure 9.

In a high-speed wind tunnel (subsonic or supersonic) assuming isentropic expansion from a total pressure H, and using equation A-2 for the variation of viscosity with temperature, the Reynolds number per unit reference length is given by

$$\frac{Re}{l} = \frac{H\,M}{\mu_a} \sqrt{\frac{\gamma}{R\,T_a}} \left(\frac{T_a}{T}\right)^{\frac{\gamma-2}{\gamma-1}} \frac{\frac{T}{T_a} + \frac{216}{T_a}}{1 + \frac{216}{T_a}} \qquad \text{(B-3)}$$

Using a constant value of H, the Reynolds number per foot has been plotted in figure 10 as a function of M for various temperatures T_a.

APPENDIX C

HUMIDITY RELATIONS

The following relationships are presented for the humidity, density, and vapor pressure:

Specific humidity of air;

$$s = \frac{p_v}{1.61\ (H-p_v)} \sim \frac{p_v}{1.61\ H} \qquad \text{(C-1)}$$

Relative humidity of air;

$$r = \frac{p_v}{p_d} \qquad \text{(C-2)}$$

$$s = 0.622\ \frac{p_d}{(H-p_v)}\ r \approx 0.622\ \frac{p_d}{H}\ r \qquad \text{(C-3)}$$

Density of air;

$$w = g\rho = \frac{pg}{RT} = 2.70\ \frac{(H - p_v)\,(\text{psi})}{(460 + T_d)\,(^\circ F)}\ \frac{\text{lb}}{\text{ft}^3} \qquad \text{(C-4)}$$

Vapor pressure from psychrometric data (Apjohn)

$$p = p_w - \frac{H}{14.7\ \text{psi}} \frac{(T_d - T_w)}{90^{\circ}F} \tag{C-5}$$

where

s specific humidity
r relative humidity
H pressure of the air and vapor mixture
T_d temperature (dry-bulb) of mixture
T_w wet-bulb temperature
p_d saturated vapor pressure corresponding to T_d
p_w saturated vapor pressure corresponding to T_w
p_v saturated vapor pressure corresponding to dew-point temperature

Saturated Vapor Pressure of Ice and Water

T °F	p psi	T °F	p psi	T °F	p psi	T °F	p psi
−90	.00005	−30	.0035	30	.0808	90	.6982
−85	.00008	−25	.0046	35	.1000	95	.8152
−80	.00012	−20	.0062	40	.1217	100	.9492
−75	.00019	−15	.0082	45	.1475	105	1.1016
−70	.00024	−10	.0108	50	.1781	110	1.2748
−65	.00035	− 5	.0142	55	.2141	115	1.4709
−60	.00050	0	.0185	60	.2563	120	1.6924
−55	.00070	5	.0240	65	.3056	125	1.9420
−50	.00098	10	.0309	70	.3631	130	2.2225
−45	.00136	15	.0396	75	.4298	135	2.5370
−40	.0019	20	.0505	80	.5069	140	2.8886
−35	.0025	25	.0640	85	.5959	145	3.281

For additional values see references 8 and 9.

APPENDIX D

CONVERSION FACTORS AND CONSTANTS

Pressures

Multiply → by to obtain ↓	$\frac{\text{lb}}{\text{in}^2}$	$\frac{\text{lb}}{\text{ft}^2}$	in. H_2O	in. Hg	cm Hg	Standard atmospheres
lb/in^2	1	0.006944	0.03613	0.4912	0.1934	14.70
lb/ft^2	144	1	5.204	70.73	27.85	2117
in. H_2O	27.68	.1922	1	13.60	5.354	406.8
in. Hg	2.036	.01414	.07355	1	.3937	29.92
cm Hg	5.171	.03591	.1868	2.540	1	76.00
Standard atmospheres	.06804	.0004725	.002458	.03342	.01316	1

Multiply	by	to obtain
Miles per hour	$\frac{22}{15} = 1.467$	Feet per second
(Miles per hour)^2	2.151	(Feet per second)^2
Radians	$\frac{180}{\pi} = 57.30$	Degrees
Square meters	10.76	Square feet
Square inches	6.452	Square centimeters
Centipoises	1.45×10^{-7}	Lb sec/in^2
Pounds (avdp)	7000	Grains
Log_{10}	2.3026	ln_e

$\pi = 3.14159$

$e = 2.71828$

$c_p = 0.241$ for dry air, room temperature, atmospheric pressure

$c_v = 0.173$ for dry air, room temperature, atmospheric pressure

$R = 1718 \frac{\text{ft}^2}{\text{sec}^2\ {}^\circ\text{F}}$ for dry air

$g = 980.665 \text{ cm/sec}^2 = 32.174 \text{ ft/sec}^2$

APPENDIX E

NACA STANDARD ATMOSPHERE

Variation of Temperature, Pressure, and Density with Altitude

For altitudes up to the lower level of the isothermal atmosphere (35,332 feet), the following exact equations are applicable (reference 10):

$$T = T_{SL} - Ch \qquad \text{(Toussaint's formula)} \tag{E-1}$$

$$\frac{p}{p_{SL}} = \left(1 - \frac{h}{145366}\right)^{5.255} \tag{E-2}$$

(h in feet)

$$\frac{\rho}{\rho_{SL}} = \left(1 - \frac{h}{145366}\right)^{4.255} \tag{E-3}$$

where

- T absolute temperature
- C constant
- h altitude
- p static pressure
- ρ density

Subscript SL refers to sea-level conditions

For the English system

$T_{SL} = 518.4\ ^{\circ}R$
$C = 0.00356617$
h = altitude, feet

For the metric system

$T_{SL} = 288\ ^{\circ}K$
$C = 0.0065$
h = altitude, meters

In the isothermal atmosphere ($35,332 < h < 104,987$ ft, and $T = 392.4\ ^{\circ}R$) the following equations apply (reference 11):

$$\log_{10}\left(\frac{p_{SL}}{p}\right) = \frac{h}{122.862\ T_m} = \frac{h}{48211} - .09759 \qquad \text{(E-4)}$$

$$T_m = \frac{h}{\dfrac{h}{392.4} - 11.9900}$$

$$\frac{\rho}{\rho_{SL}} = \frac{p}{p_{SL}} \frac{T_{SL}}{T} \qquad \text{(E-5)}$$

where T_m harmonic mean temperature, $^{\circ}R$.

Equations applicable to altitudes above the upper level of the isothermal atmosphere (104,987 ft) are given in reference 12.

Viscosity Relationships

The coefficient of viscosity can be determined closely from

$$\frac{\mu}{\mu_{SL}} = \left(\frac{T}{T_{SL}}\right)^{0.76}$$

or more accurately from (see Appendix A)

$$\frac{\mu}{\mu_{SL}} = \left(\frac{T_{SL} + 216}{T + 216}\right)\left(\frac{T}{T_{SL}}\right)^{3/2}$$

$$T \text{ in } {}^{\circ}R = {}^{\circ}F + 460$$

$$T_{SL} = 518.4\ {}^{\circ}R$$

$$\mu = 0.371 \times 10^{-6}\ \frac{\text{lb sec}}{\text{ft}^2}$$

Table of Properties

Values of temperature, speed of sound, pressure, viscosity and q/M^2 are given in table V as functions of altitude for the NACA standard atmosphere.

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TABLE I.- SUBSONIC

M	p/H	ρ/ρ_a	T/T_a	a/a_a	A*/A	V_o/a^*	V_o/a_a	$V_o/\hat{V}$
.00	1.0000	1.0000	1.0000	1.0000	.00000	0	0	0
.01	.9999	1.0000	1.0000	1.0000	.01728	.01095	.01000	.00447
.02	.9997	.9998	.9999	1.0000	.03455	.02191	.02000	.00894
.03	.9994	.9996	.9998	.9999	.05181	.03286	.03000	.01342
.04	.9989	.9992	.9997	.9998	.06905	.04381	.03999	.01789
.05	.9983	.9988	.9995	.9998	.08627	.05476	.04999	.02236
.06	.9975	.9982	.9993	.9996	.1035	.06570	.05998	.02682
.07	.9966	.9976	.9990	.9995	.1206	.07664	.06997	.03129
.08	.9955	.9968	.9987	.9994	.1377	.08758	.07995	.03575
.09	.9944	.9960	.9984	.9992	.1548	.09851	.08993	.04022
.10	.9930	.9950	.9980	.9990	.1718	.1094	.09990	.04468
.11	.9916	.9940	.9976	.9988	.1887	.1204	.1099	.04913
.12	.9900	.9928	.9971	.9986	.2056	.1313	.1198	.05359
.13	.9883	.9916	.9966	.9983	.2224	.1422	.1298	.05804
.14	.9864	.9903	.9961	.9980	.2391	.1531	.1397	.06249
.15	.9844	.9888	.9955	.9978	.2557	.1639	.1497	.06693
.16	.9823	.9873	.9949	.9974	.2723	.1748	.1596	.07137
.17	.9800	.9857	.9943	.9971	.2887	.1857	.1695	.07581
.18	.9776	.9840	.9936	.9968	.3051	.1965	.1794	.08024
.19	.9751	.9822	.9928	.9964	.3213	.2074	.1893	.08467
.20	.9725	.9803	.9921	.9960	.3374	.2182	.1992	.08909
.21	.9697	.9783	.9913	.9956	.3534	.2290	.2091	.09350
.22	.9668	.9762	.9904	.9952	.3693	.2398	.2189	.09791
.23	.9638	.9740	.9895	.9948	.3851	.2506	.2288	.1023
.24	.9607	.9718	.9886	.9943	.4007	.2614	.2386	.1067
.25	.9575	.9694	.9877	.9938	.4162	.2722	.2485	.1111
.26	.9541	.9670	.9867	.9933	.4315	.2829	.2583	.1155
.27	.9506	.9645	.9856	.9928	.4467	.2936	.2681	.1199
.28	.9470	.9619	.9846	.9923	.4618	.3043	.2778	.1242
.29	.9433	.9592	.9835	.9917	.4767	.3150	.2876	.1286
.30	.9395	.9564	.9823	.9911	.4914	.3257	.2973	.1330
.31	.9355	.9535	.9811	.9905	.5059	.3364	.3071	.1373
.32	.9315	.9506	.9799	.9899	.5203	.3470	.3168	.1417
.33	.9274	.9476	.9787	.9893	.5345	.3576	.3265	.1460
.34	.9231	.9445	.9774	.9886	.5486	.3682	.3361	.1503
.35	.9188	.9413	.9761	.9880	.5624	.3788	.3458	.1546
.36	.9143	.9380	.9747	.9873	.5761	.3893	.3554	.1589
.37	.9098	.9347	.9733	.9866	.5896	.3999	.3650	.1632
.38	.9052	.9313	.9719	.9859	.6029	.4104	.3746	.1675
.39	.9004	.9278	.9705	.9851	.6160	.4209	.3842	.1718
.40	.8956	.9243	.9690	.9844	.6289	.4313	.3937	.1761
.41	.8907	.9207	.9675	.9836	.6416	.4418	.4033	.1804
.42	.8857	.9170	.9659	.9828	.6541	.4522	.4128	.1846
.43	.8807	.9132	.9643	.9820	.6663	.4626	.4222	.1888
.44	.8755	.9094	.9627	.9812	.6784	.4729	.4317	.1931
.45	.8703	.9055	.9611	.9803	.6903	.4833	.4412	.1973
.46	.8650	.9016	.9594	.9795	.7019	.4936	.4506	.2015
.47	.8596	.8976	.9577	.9786	.7134	.5038	.4599	.2057
.48	.8541	.8935	.9560	.9777	.7246	.5141	.4693	.2099
.49	.8486	.8894	.9542	.9768	.7356	.5243	.4786	.2141
.50	.8430	.8852	.9524	.9759	.7464	.5345	.4879	.2182

M	p/H	ρ/ρ_a	T/T_a	a/a_a	A*/A	V_o/a^*	V_o/a_a	$V_o/\hat{V}$
.50	.8430	.8852	.9524	.9759	.7464	.5345	.4879	.2182
.51	.8374	.8809	.9506	.9750	.7569	.5447	.4972	.2224
.52	.8317	.8766	.9487	.9740	.7672	.5548	.5065	.2265
.53	.8259	.8723	.9468	.9730	.7773	.5649	.5157	.2306
.54	.8201	.8679	.9449	.9721	.7872	.5750	.5249	.2347
.55	.8142	.8634	.9430	.9711	.7968	.5851	.5341	.2388
.56	.8082	.8589	.9410	.9701	.8063	.5951	.5432	.2429
.57	.8022	.8544	.9390	.9690	.8155	.6051	.5523	.2470
.58	.7962	.8498	.9370	.9680	.8244	.6150	.5614	.2511
.59	.7901	.8451	.9349	.9669	.8331	.6249	.5705	.2551
.60	.7840	.8405	.9328	.9658	.8416	.6348	.5795	.2592
.61	.7778	.8357	.9307	.9647	.8499	.6447	.5885	.2632
.62	.7716	.8310	.9286	.9636	.8579	.6545	.5975	.2672
.63	.7654	.8262	.9265	.9625	.8657	.6643	.6064	.2712
.64	.7591	.8213	.9243	.9614	.8732	.6740	.6153	.2752
.65	.7528	.8164	.9221	.9603	.8806	.6837	.6242	.2791
.66	.7465	.8115	.9199	.9591	.8877	.6934	.6330	.2831
.67	.7401	.8066	.9176	.9579	.8945	.7031	.6418	.2870
.68	.7338	.8016	.9153	.9567	.9012	.7127	.6506	.2909
.69	.7274	.7966	.9131	.9555	.9076	.7223	.6593	.2949
.70	.7209	.7916	.9107	.9543	.9138	.7318	.6680	.2988
.71	.7145	.7865	.9084	.9531	.9197	.7413	.6767	.3026
.72	.7080	.7814	.9061	.9519	.9254	.7508	.6853	.3065
.73	.7016	.7763	.9037	.9506	.9309	.7602	.6940	.3103
.74	.6951	.7712	.9013	.9494	.9362	.7696	.7025	.3142
.75	.6886	.7660	.8989	.9481	.9412	.7789	.7111	.3180
.76	.6821	.7609	.8964	.9468	.9461	.7883	.7196	.3218
.77	.6756	.7557	.8940	.9455	.9507	.7975	.7280	.3256
.78	.6690	.7505	.8915	.9442	.9551	.8068	.7365	.3294
.79	.6625	.7452	.8890	.9429	.9592	.8160	.7449	.3331
.80	.6560	.7400	.8865	.9416	.9632	.8251	.7532	.3369
.81	.6495	.7347	.8840	.9402	.9669	.8343	.7616	.3406
.82	.6430	.7295	.8815	.9389	.9704	.8433	.7699	.3443
.83	.6365	.7242	.8789	.9375	.9737	.8524	.7781	.3480
.84	.6300	.7189	.8763	.9361	.9769	.8614	.7863	.3517
.85	.6235	.7136	.8737	.9347	.9797	.8704	.7945	.3553
.86	.6170	.7083	.8711	.9333	.9824	.8793	.8027	.3590
.87	.6106	.7030	.8685	.9319	.9849	.8882	.8108	.3626
.88	.6041	.6977	.8659	.9305	.9872	.8970	.8189	.3662
.89	.5977	.6924	.8632	.9291	.9893	.9058	.8269	.3698
.90	.5913	.6870	.8606	.9277	.9912	.9146	.8349	.3734
.91	.5849	.6817	.8579	.9262	.9929	.9233	.8429	.3769
.92	.5785	.6764	.8552	.9248	.9944	.9320	.8508	.3805
.93	.5721	.6711	.8525	.9233	.9958	.9407	.8587	.3840
.94	.5658	.6658	.8498	.9218	.9969	.9493	.8665	.3875
.95	.5595	.6604	.8471	.9204	.9979	.9578	.8744	.3910
.96	.5532	.6551	.8444	.9189	.9986	.9663	.8821	.3945
.97	.5469	.6498	.8416	.9174	.9992	.9748	.8899	.3980
.98	.5407	.6445	.8389	.9159	.9997	.9833	.8976	.4014
.99	.5345	.6392	.8361	.9144	.9999	.9916	.9052	.4048
1.00	.5283	.6339	.8333	.9129	1.0000	1.0000	.9129	.4082

M	M^2	p/H	ρ/ρ_a	T/T_a	a/a_a	A^*/A	q/p	q/H	β	$1/\beta$	υ	α_m
1.00	1.0000	.5283	.6339	.8333	.9129	1.0000	.7000	.3698	0	∞	0	90.00
1.01	1.0201	.5221	.6287	.8306	.9113	.9999	.7141	.3728	.1418	7.053	.04473	81.93
1.02	1.0404	.5160	.6234	.8278	.9098	.9997	.7283	.3758	.2010	4.975	.1257	78.64
1.03	1.0609	.5099	.6181	.8250	.9083	.9993	.7426	.3787	.2468	4.052	.2294	76.14
1.04	1.0816	.5039	.6129	.8222	.9067	.9987	.7571	.3815	.2857	3.501	.3510	74.06
1.05	1.1025	.4979	.6077	.8193	.9052	.9980	.7718	.3842	.3202	3.123	.4874	72.25
1.06	1.1236	.4919	.6024	.8165	.9036	.9971	.7865	.3869	.3516	2.844	.6367	70.63
1.07	1.1449	.4860	.5972	.8137	.9020	.9961	.8014	.3895	.3807	2.627	.7973	69.16
1.08	1.1664	.4800	.5920	.8108	.9005	.9949	.8165	.3919	.4079	2.451	.9680	67.81
1.09	1.1881	.4742	.5869	.8080	.8989	.9936	.8317	.3944	.4337	2.306	1.148	66.55
1.10	1.2100	.4684	.5817	.8052	.8973	.9921	.8470	.3967	.4583	2.182	1.336	65.38
1.11	1.2321	.4626	.5766	.8023	.8957	.9905	.8625	.3990	.4818	2.076	1.532	64.28
1.12	1.2544	.4568	.5714	.7994	.8941	.9888	.8781	.4011	.5044	1.983	1.735	63.23
1.13	1.2769	.4511	.5663	.7966	.8925	.9870	.8938	.4032	.5262	1.900	1.944	62.25
1.14	1.2[illegible]	.4455	.5612	.7937	.8909	.9850	.9097	.4052	.5474	1.827	2.160	61.31
1.15	1.3225	.4398	.5562	.7908	.8893	.9828	.9258	.4072	.5679	1.761	2.381	60.41
1.16	1.3456	.4343	.5511	.7879	.8877	.9806	.9419	.4090	.5879	1.701	2.607	59.55
1.17	1.3689	.4287	.5461	.7851	.8860	.9782	.9582	.4108	.6074	1.646	2.839	58.73
1.18	1.3924	.4232	.5411	.7822	.8844	.9758	.9747	.4125	.6264	1.596	3.074	57.94
1.19	1.4161	.4178	.5361	.7793	.8828	.9732	.9913	.4141	.6451	1.550	3.314	57.18
1.20	1.4400	.4124	.5311	.7764	.8811	.9705	1.008	.4157	.6633	1.508	3.558	56.44
1.21	1.4641	.4070	.5262	.7735	.8795	.9676	1.025	.4171	.6812	1.468	3.806	55.74
1.22	1.4884	.4017	.5213	.7706	.8778	.9647	1.042	.4185	.6989	1.431	4.057	55.05
1.23	1.5129	.3964	.5164	.7677	.8762	.9617	1.059	.4198	.7162	1.396	4.312	54.39
1.24	1.5376	.3912	.5115	.7648	.8745	.9586	1.076	.4211	.7332	1.364	4.569	53.75
1.25	1.5625	.3861	.5067	.7619	.8729	.9553	1.094	.4223	.7500	1.333	4.830	53.13
1.26	1.5876	.3809	.5019	.7590	.8712	.9520	1.111	.4233	.7666	1.305	5.093	52.53
1.27	1.6129	.3759	.4971	.7561	.8695	.9486	1.129	.4244	.7829	1.277	5.359	51.94
1.28	1.6384	.3708	.4923	.7532	.8679	.9451	1.147	.4253	.7990	1.252	5.627	51.38
1.29	1.6641	.3658	.4876	.7503	.8662	.9415	1.165	.4262	.8149	1.227	5.898	50.82
1.30	1.6900	.3609	.4829	.7474	.8645	.9378	1.183	.4270	.8307	1.204	6.170	50.28
1.31	1.7161	.3560	.4782	.7445	.8628	.9341	1.201	.4277	.8462	1.182	6.445	49.76
1.32	1.7424	.3512	.4736	.7416	.8611	.9302	1.220	.4283	.8616	1.161	6.721	49.25
1.33	1.7689	.3464	.4690	.7387	.8595	.9263	1.238	.4289	.8769	1.140	7.000	48.75
1.34	1.7956	.3417	.4644	.7358	.8578	.9223	1.257	.4294	.8920	1.121	7.279	48.27
1.35	1.8225	.3370	.4598	.7329	.8561	.9182	1.276	.4299	.9069	1.103	7.561	47.79
1.36	1.8496	.3323	.4553	.7300	.8544	.9141	1.295	.4303	.9217	1.085	7.844	47.33
1.37	1.8769	.3277	.4508	.7271	.8527	.9099	1.314	.4306	.9364	1.068	8.128	46.88
1.38	1.9044	.3232	.4463	.7242	.8510	.9056	1.333	.4308	.9510	1.052	8.413	46.44
1.39	1.9321	.3187	.4418	.7213	.8493	.9013	1.352	.4310	.9655	1.036	8.699	46.01
1.40	1.9600	.3142	.4374	.7184	.8476	.8969	1.372	.4311	.9798	1.021	8.987	45.58
1.41	1.9881	.3098	.4330	.7155	.8459	.8925	1.392	.4312	.9940	1.006	9.276	45.17
1.42	2.0164	.3055	.4287	.7126	.8442	.8880	1.411	.4312	1.008	.9919	9.565	44.77
1.43	2.0449	.3012	.4244	.7097	.8425	.8834	1.431	.4311	1.022	.9783	9.855	44.37
1.44	2.0736	.2969	.4201	.7069	.8407	.8788	1.452	.4310	1.036	.9651	10.15	43.98
1.45	2.1025	.2927	.4158	.7040	.8390	.8742	1.472	.4308	1.050	.9524	10.44	43.60
1.46	2.1316	.2886	.4116	.7011	.8373	.8695	1.492	.4306	1.064	.9401	10.73	43.23
1.47	2.1609	.2845	.4074	.6982	.8356	.8647	1.513	.4303	1.077	.9281	11.02	42.86
1.48	2.1904	.2804	.4032	.6954	.8339	.8599	1.533	.4299	1.091	.9165	11.32	42.51
1.49	2.2201	.2764	.3991	.6925	.8322	.8551	1.554	.4295	1.105	.9053	11.61	42.16
1.50	2.2500	.2724	.3950	.6897	.8305	.8502	1.575	.4290	1.118	.8944	11.91	41.81
1.51	2.2801	.2685	.3909	.6868	.8287	.8453	1.596	.4285	1.131	.8838	12.20	41.47
1.52	2.3104	.2646	.3869	.6840	.8270	.8404	1.617	.4279	1.145	.8736	12.49	41.14
1.53	2.3409	.2608	.3829	.6811	.8253	.8354	1.639	.4273	1.158	.8636	12.79	40.81
1.54	2.3716	.2570	.3789	.6783	.8236	.8304	1.660	.4266	1.171	.8539	13.09	40.49
1.55	2.4025	.2533	.3750	.6754	.8219	.8254	1.682	.4259	1.184	.8444	13.38	40.18
1.56	2.4336	.2496	.3710	.6726	.8201	.8203	1.704	.4252	1.197	.8352	13.68	39.87
1.57	2.4649	.2459	.3672	.6698	.8184	.8152	1.725	.4243	1.210	.8262	13.97	39.56
1.58	2.4964	.2423	.3633	.6670	.8167	.8101	1.747	.4235	1.223	.8175	14.27	39.27
1.59	2.5281	.2388	.3595	.6642	.8150	.8050	1.770	.4226	1.236	.8090	14.56	38.97
1.60	2.5600	.2353	.3557	.6614	.8133	.7998	1.792	.4216	1.249	.8006	14.86	38.68
1.61	2.5921	.2318	.3520	.6586	.8115	.7947	1.814	.4206	1.262	.7925	15.16	38.40
1.62	2.6244	.2284	.3483	.6558	.8098	.7895	1.837	.4196	1.275	.7846	15.45	38.12
1.63	2.6569	.2250	.3446	.6530	.8081	.7843	1.860	.4185	1.287	.7769	15.75	37.84
1.64	2.6896	.2217	.3409	.6502	.8064	.7791	1.883	.4174	1.300	.7693	16.04	37.57
1.65	2.7225	.2184	.3373	.6475	.8046	.7739	1.906	.4162	1.312	.7619	16.34	37.31
1.66	2.7556	.2151	.3337	.6447	.8029	.7686	1.929	.4150	1.325	.7547	16.63	37.04
1.67	2.7889	.2119	.3302	.6419	.8012	.7634	1.952	.4138	1.337	.7477	16.93	36.78
1.68	2.8224	.2088	.3266	.6392	.7995	.7581	1.976	.4125	1.350	.7408	17.22	36.53
1.69	2.8561	.2057	.3232	.6364	.7978	.7529	1.999	.4112	1.362	.7340	17.52	36.28
1.70	2.8900	.2026	.3197	.6337	.7961	.7476	2.023	.4098	1.375	.7274	17.81	36.03

NATIONAL ADVISORY

M	M^2	p/H	ρ/ρ_a	T/T_a	a/a_a	A^*/A	q/p	q/H	β	$1/\beta$	υ	α_m
1.70	2.8900	.2026	.3197	.6337	.7961	.7476	2.023	.4098	1.375	.7274	17.81	36.03
1.71	2.9241	.1996	.3163	.6310	.7943	.7423	2.047	.4086	1.387	.7209	18.10	35.79
1.72	2.9584	.1966	.3129	.6283	.7926	.7371	2.071	.4071	1.399	.7146	18.40	35.55
1.73	2.9929	.1936	.3095	.6256	.7909	.7318	2.095	.4056	1.412	.7084	18.69	35.31
1.74	3.0276	.1907	.3062	.6229	.7892	.7265	2.119	.4041	1.424	.7023	18.98	35.08
1.75	3.0625	.1878	.3029	.6202	.7875	.7212	2.144	.4026	1.436	.6963	19.27	34.85
1.76	3.0976	.1850	.2996	.6175	.7858	.7160	2.168	.4011	1.448	.6905	19.56	34.62
1.77	3.1329	.1822	.2964	.6148	.7841	.7107	2.193	.3996	1.460	.6847	19.86	34.40
1.78	3.1684	.1794	.2932	.6121	.7824	.7054	2.218	.3980	1.473	.6791	20.15	34.18
1.79	3.2041	.1767	.2900	.6095	.7807	.7002	2.243	.3964	1.485	.6736	20.44	33.96
1.80	3.2400	.1740	.2868	.6068	.7790	.6949	2.268	.3947	1.497	.6682	20.73	33.75
1.81	3.2761	.1714	.2837	.6041	.7773	.6897	2.293	.3931	1.509	.6628	21.01	33.54
1.82	3.3124	.1688	.2806	.6015	.7756	.6845	2.319	.3914	1.521	.6576	21.30	33.33
1.83	3.3489	.1662	.2776	.5989	.7739	.6792	2.344	.3897	1.533	.6525	21.59	33.12
1.84	3.3856	.1637	.2745	.5963	.7722	.6740	2.370	.3879	1.545	.6474	21.88	32.92
1.85	3.4225	.1612	.2715	.5936	.7705	.6688	2.396	.3862	1.556	.6425	22.16	32.72
1.86	3.4596	.1587	.2686	.5910	.7688	.6636	2.422	.3844	1.568	.6376	22.45	32.52
1.87	3.4969	.1563	.2656	.5884	.7671	.6584	2.448	.3826	1.580	.6328	22.73	32.33
1.88	3.5344	.1539	.2627	.5859	.7654	.6533	2.474	.3808	1.592	.6281	23.02	32.13
1.89	3.5721	.1516	.2598	.5833	.7637	.6481	2.500	.3790	1.604	.6235	23.30	31.94
1.90	3.6100	.1492	.2570	.5807	.7620	.6430	2.527	.3771	1.616	.6190	23.59	31.76
1.91	3.6481	.1470	.2542	.5782	.7604	.6379	2.554	.3753	1.627	.6145	23.87	31.57
1.92	3.6864	.1447	.2514	.5756	.7587	.6328	2.580	.3734	1.639	.6101	24.15	31.39
1.93	3.7249	.1425	.2486	.5731	.7570	.6277	2.607	.3715	1.651	.6058	24.43	31.21
1.94	3.7636	.1403	.2459	.5705	.7553	.6226	2.635	.3696	1.662	.6015	24.71	31.03
1.95	3.8025	.1381	.2432	.5680	.7537	.6175	2.662	.3677	1.674	.5973	24.99	30.85
1.96	3.8416	.1360	.2405	.5655	.7520	.6125	2.689	.3657	1.686	.5932	25.27	30.68
1.97	3.8809	.1339	.2378	.5630	.7503	.6075	2.717	.3638	1.697	.5892	25.55	30.51
1.98	3.9204	.1318	.2352	.5605	.7487	.6025	2.744	.3618	1.709	.5852	25.83	30.33
1.99	3.9601	.1298	.2326	.5580	.7470	.5975	2.772	.3598	1.720	.5812	26.10	30.17
2.00	4.0000	.1278	.2300	.5556	.7454	.5926	2.800	.3579	1.732	.5774	26.38	30.00
2.01	4.0401	.1258	.2275	.5531	.7437	.5877	2.828	.3559	1.744	.5735	26.66	29.84
2.02	4.0804	.1239	.2250	.5506	.7420	.5828	2.856	.3539	1.755	.5698	26.93	29.67
2.03	4.1209	.1220	.2225	.5482	.7404	.5779	2.885	.3518	1.767	.5661	27.20	29.51
2.04	4.1616	.1201	.2200	.5458	.7388	.5730	2.913	.3498	1.778	.5624	27.48	29.35
2.05	4.2025	.1182	.2176	.5433	.7371	.5682	2.942	.3478	1.790	.5588	27.75	29.20
2.06	4.2436	.1164	.2152	.5409	.7355	.5634	2.971	.3458	1.801	.5552	28.02	29.04
2.07	4.2849	.1146	.2128	.5385	.7338	.5586	2.999	.3437	1.812	.5517	28.29	28.89
2.08	4.3264	.1128	.2104	.5361	.7322	.5538	3.028	.3417	1.824	.5483	28.56	28.74
2.09	4.3681	.1111	.2081	.5337	.7306	.5491	3.058	.3396	1.835	.5449	28.83	28.59
2.10	4.4100	.1094	.2058	.5313	.7289	.5444	3.087	.3376	1.847	.5415	29.10	28.44
2.11	4.4521	.1077	.2035	.5290	.7273	.5397	3.116	.3355	1.858	.5382	29.36	28.29
2.12	4.4944	.1060	.2013	.5266	.7257	.5350	3.146	.3334	1.869	.5350	29.63	28.14
2.13	4.5369	.1043	.1990	.5243	.7241	.5304	3.176	.3314	1.881	.5317	29.90	28.00
2.14	4.5796	.1027	.1968	.5219	.7225	.5258	3.206	.3293	1.892	.5285	30.16	27.86
2.15	4.6225	.1011	.1946	.5196	.7208	.5212	3.236	.3272	1.903	.5254	30.43	27.72
2.16	4.6656	.09956	.1925	.5173	.7192	.5167	3.266	.3252	1.915	.5223	30.69	27.58
2.17	4.7089	.09802	.1903	.5150	.7176	.5122	3.296	.3231	1.926	.5193	30.95	27.44
2.18	4.7524	.09650	.1882	.5127	.7160	.5077	3.327	.3210	1.937	.5162	31.21	27.30
2.19	4.7961	.09500	.1861	.5104	.7144	.5032	3.357	.3189	1.948	.5133	31.47	27.17
2.20	4.8400	.09352	.1841	.5081	.7128	.4988	3.388	.3169	1.960	.5103	31.73	27.04
2.21	4.8841	.09207	.1820	.5059	.7112	.4944	3.419	.3148	1.971	.5074	31.99	26.90
2.22	4.9284	.09064	.1800	.5036	.7097	.4900	3.450	.3127	1.982	.5045	32.25	26.77
2.23	4.9729	.08923	.1780	.5014	.7081	.4856	3.481	.3106	1.993	.5017	32.51	26.64
2.24	5.0176	.08785	.1760	.4991	.7065	.4813	3.512	.3085	2.004	.4989	32.76	26.51
2.25	5.0625	.08648	.1740	.4969	.7049	.4770	3.544	.3065	2.016	.4961	33.02	26.39
2.26	5.1076	.08514	.1721	.4947	.7033	.4727	3.575	.3044	2.027	.4934	33.27	26.26
2.27	5.1529	.08382	.1702	.4925	.7018	.4685	3.607	.3023	2.038	.4907	33.53	26.14
2.28	5.1984	.08252	.1683	.4903	.7002	.4643	3.639	.3003	2.049	.4880	33.78	26.01
2.29	5.2441	.08123	.1664	.4881	.6986	.4601	3.671	.2982	2.060	.4854	34.03	25.89
2.30	5.2900	.07997	.1646	.4859	.6971	.4560	3.703	.2961	2.071	.4828	34.28	25.77
2.31	5.3361	.07873	.1628	.4837	.6955	.4519	3.735	.2941	2.082	.4802	34.53	25.65
2.32	5.3824	.07751	.1609	.4816	.6940	.4478	3.768	.2920	2.093	.4777	34.78	25.53
2.33	5.4289	.07631	.1592	.4794	.6924	.4437	3.800	.2900	2.104	.4752	35.03	25.42
2.34	5.4756	.07512	.1574	.4773	.6909	.4397	3.833	.2879	2.116	.4727	35.28	25.30
2.35	5.5225	.07396	.1556	.4752	.6893	.4357	3.866	.2859	2.127	.4702	35.53	25.18
2.36	5.5696	.07281	.1539	.4731	.6878	.4317	3.899	.2839	2.138	.4678	35.77	25.07
2.37	5.6169	.07168	.1522	.4709	.6863	.4278	3.932	.2818	2.149	.4654	36.02	24.96
2.38	5.6644	.07057	.1505	.4688	.6847	.4239	3.965	.2798	2.160	.4630	36.26	24.85
2.39	5.7121	.06948	.1488	.4668	.6832	.4200	3.998	.2778	2.171	.4607	36.50	24.73
2.40	5.7600	.06840	.1472	.4647	.6817	.4161	4.032	.2758	2.182	.4583	36.75	24.62

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE II.- CONTINUED. SUPERSONIC

M	M^2	p/H	ρ/ρ_a	T/T_a	a/a_a	A*/A	q/p	q/H	β	$1/\beta$	υ	α_m
2.40	5.7600	.06840	.1472	.4647	.6817	.4161	4.032	.2758	2.182	.4583	36.75	24.62
2.41	5.8081	.06734	.1456	.4626	.6802	.4123	4.066	.2738	2.193	.4561	36.99	24.52
2.42	5.8564	.06630	.1439	.4606	.6786	.4085	4.099	.2718	2.204	.4538	37.23	24.41
2.43	5.9049	.06527	.1424	.4585	.6771	.4048	4.133	.2698	2.215	.4515	37.47	24.30
2.44	5.9536	.06426	.1408	.4565	.6756	.4010	4.168	.2678	2.226	.4493	37.71	24.19
2.45	6.0025	.06327	.1392	.4544	.6741	.3973	4.202	.2658	2.237	.4471	37.95	24.09
2.46	6.0516	.06229	.1377	.4524	.6726	.3937	4.236	.2639	2.248	.4449	38.18	23.99
2.47	6.1009	.06133	.1362	.4504	.6711	.3900	4.271	.2619	2.259	.4428	38.42	23.88
2.48	6.1504	.06038	.1347	.4484	.6696	.3864	4.305	.2599	2.269	.4406	38.66	23.78
2.49	6.2001	.05945	.1332	.4464	.6681	.3828	4.340	.2580	2.280	.4385	38.89	23.68
2.50	6.2500	.05853	.1317	.4444	.6667	.3793	4.375	.2561	2.291	.4364	39.12	23.58
2.51	6.3001	.05762	.1302	.4425	.6652	.3757	4.410	.2541	2.302	.4344	39.36	23.48
2.52	6.3504	.05674	.1288	.4405	.6637	.3722	4.445	.2522	2.313	.4323	39.59	23.38
2.53	6.4009	.05586	.1274	.4386	.6622	.3688	4.481	.2503	2.324	.4303	39.82	23.28
2.54	6.4516	.05500	.1260	.4366	.6608	.3653	4.516	.2484	2.335	.4283	40.05	23.18
2.55	6.5025	.05415	.1246	.4347	.6593	.3619	4.552	.2465	2.346	.4263	40.28	23.09
2.56	6.5536	.05332	.1232	.4328	.6579	.3585	4.588	.2446	2.357	.4243	40.51	22.99
2.57	6.6049	.05250	.1218	.4309	.6564	.3552	4.623	.2427	2.367	.4224	40.75	22.91
2.58	6.6564	.05169	.1205	.4289	.6549	.3519	4.659	.2409	2.378	.4205	40.96	22.81
2.59	6.7081	.05090	.1192	.4271	.6535	.3486	4.696	.2390	2.389	.4186	41.19	22.71
2.60	6.7600	.05012	.1179	.4252	.6521	.3453	4.732	.2371	2.400	.4167	41.41	22.62
2.61	6.8121	.04935	.1166	.4233	.6506	.3421	4.768	.2353	2.411	.4148	41.64	22.53
2.62	6.8644	.04859	.1153	.4214	.6492	.3389	4.805	.2335	2.422	.4129	41.86	22.44
2.63	6.9169	.04784	.1140	.4196	.6477	.3357	4.842	.2317	2.432	.4111	42.09	22.35
2.64	6.9696	.04711	.1128	.4177	.6463	.3325	4.879	.2298	2.443	.4093	42.31	22.26
2.65	7.0225	.04639	.1115	.4159	.6449	.3294	4.916	.2280	2.454	.4075	42.53	22.17
2.66	7.0756	.04568	.1103	.4141	.6435	.3263	4.953	.2262	2.465	.4057	42.75	22.08
2.67	7.1289	.04498	.1091	.4122	.6421	.3232	4.990	.2245	2.476	.4039	42.97	22.00
2.68	7.1824	.04429	.1079	.4104	.6406	.3202	5.028	.2227	2.486	.4022	43.19	21.91
2.69	7.2361	.04362	.1067	.4086	.6392	.3172	5.065	.2209	2.497	.4004	43.40	21.82
2.70	7.2900	.04295	.1056	.4068	.6378	.3142	5.103	.2192	2.508	.3987	43.62	21.74
2.71	7.3441	.04229	.1044	.4051	.6364	.3112	5.141	.2174	2.519	.3970	43.84	21.65
2.72	7.3984	.04165	.1033	.4033	.6350	.3083	5.179	.2157	2.530	.3953	44.05	21.57
2.73	7.4529	.04102	.1022	.4015	.6337	.3054	5.217	.2140	2.540	.3937	44.27	21.49
2.74	7.5076	.04039	.1010	.3998	.6323	.3025	5.255	.2123	2.551	.3920	44.48	21.41
2.75	7.5625	.03978	.09994	.3980	.6309	.2996	5.294	.2106	2.562	.3904	44.69	21.32
2.76	7.6176	.03917	.09885	.3963	.6295	.2968	5.332	.2089	2.572	.3887	44.91	21.24
2.77	7.6729	.03858	.09778	.3945	.6281	.2940	5.371	.2072	2.583	.3871	45.12	21.16
2.78	7.7284	.03799	.09671	.3928	.6268	.2912	5.410	.2055	2.594	.3855	45.33	21.08
2.79	7.7841	.03742	.09566	.3911	.6254	.2884	5.449	.2039	2.605	.3839	45.54	21.00
2.80	7.8400	.03685	.09463	.3894	.6240	.2857	5.488	.2022	2.615	.3824	45.75	20.92
2.81	7.8961	.03629	.09360	.3877	.6227	.2830	5.527	.2006	2.626	.3808	45.95	20.85
2.82	7.9524	.03574	.09259	.3860	.6213	.2803	5.567	.1990	2.637	.3793	46.16	20.77
2.83	8.0089	.03520	.09158	.3844	.6200	.2777	5.606	.1973	2.647	.3777	46.37	20.69
2.84	8.0656	.03467	.09059	.3827	.6186	.2750	5.646	.1957	2.658	.3762	46.57	20.62
2.85	8.1225	.03415	.08962	.3810	.6173	.2724	5.686	.1941	2.669	.3747	46.78	20.54
2.86	8.1796	.03363	.08865	.3794	.6159	.2698	5.726	.1926	2.679	.3732	46.98	20.47
2.87	8.2369	.03312	.08769	.3777	.6146	.2673	5.766	.1910	2.690	.3717	47.19	20.39
2.88	8.2944	.03263	.08675	.3761	.6133	.2648	5.806	.1894	2.701	.3703	47.39	20.32
2.89	8.3521	.03213	.08581	.3745	.6119	.2622	5.846	.1879	2.711	.3688	47.59	20.24
2.90	8.4100	.03165	.08489	.3729	.6106	.2598	5.887	.1863	2.722	.3674	47.79	20.17
2.91	8.4681	.03118	.08398	.3712	.6093	.2573	5.928	.1848	2.733	.3659	47.99	20.10
2.92	8.5264	.03071	.08307	.3696	.6080	.2549	5.968	.1833	2.743	.3645	48.19	20.03
2.93	8.5849	.03025	.08218	.3681	.6067	.2524	6.009	.1818	2.754	.3631	48.39	19.96
2.94	8.6436	.02980	.08130	.3665	.6054	.2500	6.051	.1803	2.765	.3617	48.59	19.89
2.95	8.7025	.02935	.08043	.3649	.6041	.2477	6.092	.1788	2.775	.3603	48.78	19.81
2.96	8.7616	.02891	.07957	.3633	.6028	.2453	6.133	.1773	2.786	.3589	48.98	19.75
2.97	8.8209	.02848	.07872	.3618	.6015	.2430	6.175	.1758	2.797	.3576	49.18	19.68
2.98	8.8804	.02805	.07788	.3602	.6002	.2407	6.216	.1744	2.807	.3562	49.37	19.61
2.99	8.9401	.02764	.07705	.3587	.5989	.2384	6.258	.1729	2.818	.3549	49.56	19.54
3.00	9.0000	.02722	.07623	.3571	.5976	.2362	6.300	.1715	2.828	.3536	49.76	19.47
3.01	9.0601	.02682	.07541	.3556	.5963	.2339	6.342	.1701	2.839	.3522	49.95	19.40
3.02	9.1204	.02642	.07461	.3541	.5951	.2317	6.384	.1687	2.850	.3509	50.14	19.34
3.03	9.1809	.02603	.07382	.3526	.5938	.2295	6.427	.1673	2.860	.3496	50.33	19.27
3.04	9.2416	.02564	.07303	.3511	.5925	.2273	6.469	.1659	2.871	.3483	50.52	19.20
3.05	9.3025	.02526	.07226	.3496	.5913	.2252	6.512	.1645	2.881	.3471	50.71	19.14
3.06	9.3636	.02489	.07149	.3481	.5900	.2230	6.555	.1631	2.892	.3458	50.90	19.07
3.07	9.4249	.02452	.07074	.3466	.5887	.2209	6.597	.1618	2.903	.3445	51.09	19.01
3.08	9.4864	.02416	.06999	.3452	.5875	.2188	6.640	.1604	2.913	.3433	51.28	18.95
3.09	9.5481	.02380	.06925	.3437	.5862	.2168	6.684	.1591	2.924	.3420	51.46	18.88
3.10	9.6100	.02345	.06852	.3422	.5850	.2147	6.727	.1577	2.934	.3408	51.65	18.82

TABLE II.- CONTINUED. SUPERSONIC

M	M^2	p/H	ρ/ρ_a	T/T_a	a/a_a	A*/A	q/p	q/H	β	$1/\beta$	ν	α_m
3.10	9.6100	.02345	.06852	.3422	.5850	.2147	6.727	.1577	2.934	.3408	51.65	18.82
3.11	9.6721	.02310	.06779	.3408	.5838	.2127	6.770	.1564	2.945	.3396	51.84	18.76
3.12	9.7344	.02276	.06708	.3393	.5825	.2107	6.814	.1551	2.955	.3384	52.02	18.69
3.13	9.7969	.02243	.06637	.3379	.5813	.2087	6.858	.1538	2.966	.3372	52.20	18.63
3.14	9.8596	.02210	.06568	.3365	.5801	.2067	6.902	.1525	2.977	.3360	52.39	18.57
3.15	9.9225	.02177	.06499	.3351	.5788	.2048	6.946	.1512	2.987	.3348	52.57	18.51
3.16	9.9856	.02146	.06430	.3337	.5776	.2028	6.990	.1500	2.998	.3336	52.75	18.45
3.17	10.0489	.02114	.06363	.3323	.5764	.2009	7.034	.1487	3.008	.3324	52.93	18.39
3.18	10.1124	.02083	.06296	.3309	.5752	.1990	7.079	.1475	3.019	.3313	53.11	18.33
3.19	10.1761	.02053	.06231	.3295	.5740	.1971	7.123	.1462	3.029	.3301	53.29	18.27
3.20	10.2400	.02023	.06165	.3281	.5728	.1953	7.168	.1450	3.040	.3290	53.47	18.21
3.21	10.3041	.01993	.06101	.3267	.5716	.1934	7.213	.1438	3.050	.3278	53.65	18.15
3.22	10.3684	.01964	.06037	.3253	.5704	.1916	7.258	.1426	3.061	.3267	53.83	18.09
3.23	10.4329	.01936	.05975	.3240	.5692	.1898	7.303	.1414	3.071	.3256	54.00	18.03
3.24	10.4976	.01908	.05912	.3226	.5680	.1880	7.348	.1402	3.082	.3245	54.18	17.98
3.25	10.5625	.01880	.05851	.3213	.5668	.1863	7.394	.1390	3.092	.3234	54.35	17.92
3.26	10.6276	.01853	.05790	.3199	.5656	.1845	7.439	.1378	3.103	.3223	54.53	17.86
3.27	10.6929	.01826	.05730	.3186	.5645	.1828	7.485	.1367	3.113	.3212	54.71	17.81
3.28	10.7584	.01799	.05671	.3173	.5633	.1810	7.531	.1355	3.124	.3201	54.88	17.75
3.29	10.8241	.01773	.05612	.3160	.5621	.1793	7.577	.1344	3.134	.3190	55.05	17.70
3.30	10.8900	.01748	.05554	.3147	.5609	.1777	7.623	.1332	3.145	.3180	55.22	17.64
3.31	10.9561	.01722	.05497	.3134	.5598	.1760	7.669	.1321	3.155	.3169	55.39	17.58
3.32	11.0224	.01698	.05440	.3121	.5586	.1743	7.716	.1310	3.166	.3159	55.56	17.53
3.33	11.0889	.01673	.05384	.3108	.5575	.1727	7.762	.1299	3.176	.3148	55.73	17.48
3.34	11.1556	.01649	.05329	.3095	.5563	.1711	7.809	.1288	3.187	.3138	55.90	17.42
3.35	11.2225	.01625	.05274	.3082	.5552	.1695	7.856	.1277	3.197	.3128	56.07	17.37
3.36	11.2896	.01602	.05220	.3069	.5540	.1679	7.903	.1266	3.208	.3117	56.24	17.31
3.37	11.3569	.01579	.05166	.3057	.5529	.1663	7.950	.1255	3.218	.3107	56.41	17.26
3.38	11.4244	.01557	.05113	.3044	.5517	.1648	7.997	.1245	3.229	.3097	56.58	17.21
3.39	11.4921	.01534	.05061	.3032	.5506	.1632	8.044	.1234	3.239	.3087	56.75	17.16
3.40	11.5600	.01513	.05009	.3019	.5495	.1617	8.092	.1224	3.250	.3077	56.91	17.10
3.41	11.6281	.01491	.04958	.3007	.5484	.1602	8.140	.1214	3.260	.3067	57.07	17.05
3.42	11.6964	.01470	.04908	.2995	.5472	.1587	8.188	.1203	3.271	.3058	57.24	17.00
3.43	11.7649	.01449	.04858	.2982	.5461	.1572	8.235	.1193	3.281	.3048	57.40	16.95
3.44	11.8336	.01428	.04808	.2970	.5450	.1558	8.284	.1183	3.291	.3038	57.56	16.90
3.45	11.9025	.01408	.04759	.2958	.5439	.1543	8.332	.1173	3.302	.3029	57.73	16.85
3.46	11.9716	.01388	.04711	.2946	.5428	.1529	8.380	.1163	3.312	.3019	57.89	16.80
3.47	12.0409	.01368	.04663	.2934	.5417	.1515	8.429	.1153	3.323	.3010	58.05	16.75
3.48	12.1104	.01349	.04616	.2922	.5406	.1501	8.477	.1144	3.333	.3000	58.21	16.70
3.49	12.1801	.01330	.04569	.2910	.5395	.1487	8.526	.1134	3.344	.2991	58.37	16.65
3.50	12.2500	.01311	.04523	.2899	.5384	.1473	8.575	.1124	3.354	.2981	58.53	16.60
3.60	12.9600	.01138	.04089	.2784	.5276	.1342	9.072	.1033	3.458	.2892	60.09	16.13
3.70	13.6900	9.903×10^{-3}	.03702	.2675	.5172	.1224	9.583	.09490	3.562	.2807	61.60	15.68
3.80	14.4400	8.629×10^{-3}	.03355	.2572	.5072	.1117	10.11	.08722	3.666	.2728	63.04	15.26
3.90	15.2100	7.532×10^{-3}	.03044	.2474	.4974	.1021	10.65	.08019	3.770	.2653	64.44	14.86
4.00	16.0000	6.586×10^{-3}	.02766	.2381	.4880	.09329	11.20	.07376	3.873	.2582	65.78	14.48
4.10	16.8100	5.769×10^{-3}	.02516	.2293	.4788	.08536	11.77	.06788	3.976	.2515	67.08	14.12
4.20	17.6400	5.062×10^{-3}	.02292	.2208	.4699	.07818	12.35	.06251	4.079	.2451	68.33	13.77
4.30	18.4900	4.449×10^{-3}	.02090	.2129	.4614	.07166	12.94	.05759	4.182	.2391	69.54	13.45
4.40	19.3600	3.918×10^{-3}	.01909	.2053	.4531	.06575	13.55	.05309	4.285	.2334	70.71	13.14
4.50	20.2500	3.455×10^{-3}	.01745	.1980	.4450	.06038	14.18	.04898	4.387	.2279	71.83	12.84
4.60	21.1600	3.053×10^{-3}	.01597	.1911	.4372	.05550	14.81	.04521	4.490	.2227	72.92	12.56
4.70	22.0900	2.701×10^{-3}	.01464	.1846	.4296	.05107	15.46	.04177	4.592	.2178	73.97	12.28
4.80	23.0400	2.394×10^{-3}	.01343	.1783	.4223	.04703	16.13	.03861	4.695	.2130	74.99	12.02
4.90	24.0100	2.126×10^{-3}	.01233	.1724	.4152	.04335	16.81	.03572	4.797	.2085	75.97	11.78
5.00	25.0000	1.890×10^{-3}	.01134	.1667	.4082	.04000	17.50	.03308	4.899	.2041	76.92	11.54
6.00	36.0000	6.334×10^{-4}	5.194×10^{-3}	.1220	.3492	.01880	25.20	.01596	5.916	.1690	84.96	9.594
7.00	49.0000	2.416×10^{-4}	2.609×10^{-3}	.09259	.3043	9.602×10^{-3}	34.30	8.285×10^{-3}	6.928	.1443	90.97	8.213
8.00	64.0000	1.024×10^{-4}	1.414×10^{-3}	.07246	.2692	5.260×10^{-3}	44.80	4.589×10^{-3}	7.937	.1260	95.62	7.181

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TABLE II.– CONCLUDED. SUPERSONIC

M	M^2	p/H	ρ/ρ_a	T/T_a	a/a_a	A*/A	q/p	q/H	β	$1/\beta$	ν	α_m
8.00	64.0000	1.024×10^{-4}	1.414×10^{-3}	.07246	.2692	5.260×10^{-3}	44.80	4.589×10^{-3}	7.937	.1260	95.62	7.181
9.00	81.0000	4.739×10^{-5}	8.150×10^{-4}	.05814	.2411	3.056×10^{-3}	56.70	2.687×10^{-3}	8.944	.1118	99.32	6.379
10.00	100.0000	2.356×10^{-5}	4.948×10^{-4}	.04762	.2182	1.866×10^{-3}	70.00	1.649×10^{-3}	9.950	.1005	102.3	5.739
15.00	225.0000	1.515×10^{-6}	6.968×10^{-5}	.02174	.1474	2.663×10^{-4}	157.5	2.386×10^{-4}	14.97	.06682	111.5	3.823
20.00	400.0000	2.091×10^{-7}	1.694×10^{-5}	.01235	.1111	6.503×10^{-5}	280.0	5.854×10^{-5}	19.97	.05006	116.2	2.866
100.00	10^4	2.790×10^{-12}	5.583×10^{-9}	4.998×10^{-4}	.02236	2.157×10^{-8}	7000.0	1.953×10^{-8}	100.0	.01000	127.6	.5730
∞	∞	0	0	0	0	0	∞	0	∞	0	130.5	0

Definition of Symbols for Table II

M Mach number

p/H ratio of static pressure to total pressure

ρ/ρ_a ratio of local density to stagnation density

T/T_a ratio of local temperature to stagnation temperature

a/a_a ratio of local speed of sound to speed of sound at stagnation conditions

A*/A ratio of area of throat to local cross sectional area of a stream tube

q/p ratio of $\frac{1}{2}\rho V^2$ to static pressure

q/H ratio of $\frac{1}{2}\rho V^2$ to total pressure

β the factor $\sqrt{M^2-1}$

ν angle-of-turning of a supersonic stream from $M=1$ to M, degrees

α_m Mach angle, degrees

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TABLE III.- NORMAL SHOCK WAVES

M_0	M_1	p_0/H_0	p_1/p_0	ρ_1/ρ_0	T_1/T_0	a_1/a_0	H_1/H_0	p_1/H_1	p_1/H_0	V_0/a^*	V_0/a_a	$V_0/\hat{V}$
1.00	1.0000	.5283	1.000	1.000	1.000	1.000	1.0000	.5283	.5283	1.000	.9129	.4082
1.01	.9901	.5221	1.023	1.017	1.007	1.003	1.0000	.5344	.5344	1.008	.9205	.4116
1.02	.9805	.5160	1.047	1.033	1.013	1.007	1.0000	.5403	.5403	1.017	.9280	.4150
1.03	.9712	.5099	1.071	1.050	1.020	1.010	1.0000	.5462	.5462	1.025	.9355	.4184
1.04	.9620	.5039	1.095	1.067	1.026	1.013	.9999	.5519	.5519	1.033	.9430	.4217
1.05	.9531	.4979	1.120	1.084	1.033	1.016	.9999	.5575	.5574	1.041	.9504	.4250
1.06	.9444	.4919	1.144	1.101	1.039	1.019	.9998	.5630	.5629	1.049	.9578	.4284
1.07	.9360	.4860	1.169	1.118	1.046	1.023	.9996	.5683	.5681	1.057	.9652	.4316
1.08	.9277	.4800	1.194	1.135	1.052	1.026	.9994	.5736	.5732	1.065	.9725	.4349
1.09	.9196	.4742	1.219	1.152	1.059	1.029	.9992	.5787	.5782	1.073	.9798	.4382
1.10	.9118	.4684	1.245	1.169	1.065	1.032	.9989	.5837	.5831	1.081	.9870	.4414
1.11	.9041	.4626	1.271	1.186	1.071	1.035	.9986	.5886	.5878	1.089	.9942	.4446
1.12	.8966	.4568	1.297	1.203	1.078	1.038	.9982	.5935	.5924	1.097	1.001	.4478
1.13	.8892	.4511	1.323	1.221	1.084	1.041	.9978	.5982	.5968	1.105	1.009	.4510
1.14	.8820	.4455	1.350	1.238	1.090	1.044	.9973	.6028	.6012	1.113	1.016	.4542
1.15	.8750	.4398	1.376	1.255	1.097	1.047	.9967	.6073	.6053	1.120	1.023	.4574
1.16	.8682	.4343	1.403	1.272	1.103	1.050	.9961	.6118	.6093	1.128	1.030	.4605
1.17	.8615	.4287	1.430	1.290	1.109	1.053	.9953	.6161	.6132	1.136	1.037	.4636
1.18	.8549	.4232	1.458	1.307	1.115	1.056	.9946	.6203	.6170	1.143	1.044	.4667
1.19	.8485	.4178	1.485	1.324	1.122	1.059	.9937	.6245	.6206	1.151	1.051	.4698
1.20	.8422	.4124	1.513	1.342	1.128	1.062	.9928	.6286	.6241	1.158	1.057	.4729
1.21	.8360	.4070	1.541	1.359	1.134	1.065	.9918	.6326	.6274	1.166	1.064	.4759
1.22	.8300	.4017	1.570	1.376	1.141	1.068	.9907	.6365	.6306	1.173	1.071	.4790
1.23	.8241	.3964	1.598	1.394	1.147	1.071	.9896	.6403	.6337	1.181	1.078	.4820
1.24	.8183	.3912	1.627	1.411	1.153	1.074	.9884	.6441	.6366	1.188	1.084	.4850
1.25	.8126	.3861	1.656	1.429	1.159	1.077	.9871	.6478	.6394	1.195	1.091	.4880
1.26	.8071	.3809	1.686	1.446	1.166	1.080	.9857	.6514	.6421	1.202	1.098	.4909
1.27	.8016	.3759	1.715	1.463	1.172	1.083	.9842	.6549	.6446	1.210	1.104	.4939
1.28	.7963	.3708	1.745	1.481	1.178	1.085	.9827	.6584	.6470	1.217	1.111	.4968
1.29	.7911	.3658	1.775	1.498	1.185	1.088	.9811	.6618	.6493	1.224	1.117	.4997
1.30	.7860	.3609	1.805	1.516	1.191	1.091	.9794	.6652	.6514	1.231	1.124	.5026
1.31	.7809	.3560	1.835	1.533	1.197	1.094	.9776	.6684	.6535	1.238	1.130	.5055
1.32	.7760	.3512	1.866	1.551	1.204	1.097	.9758	.6717	.6554	1.245	1.137	.5084
1.33	.7712	.3464	1.897	1.568	1.210	1.100	.9738	.6748	.6571	1.252	1.143	.5112
1.34	.7664	.3417	1.928	1.585	1.216	1.103	.9718	.6779	.6588	1.259	1.149	.5140
1.35	.7618	.3370	1.960	1.603	1.223	1.106	.9697	.6809	.6603	1.266	1.156	.5168
1.36	.7572	.3323	1.991	1.620	1.229	1.109	.9676	.6839	.6617	1.273	1.162	.5196
1.37	.7527	.3277	2.023	1.638	1.235	1.111	.9653	.6868	.6630	1.280	1.168	.5224
1.38	.7483	.3232	2.055	1.655	1.242	1.114	.9630	.6897	.6642	1.286	1.174	.5252
1.39	.7440	.3187	2.087	1.672	1.248	1.117	.9606	.6925	.6652	1.293	1.181	.5279
1.40	.7397	.3142	2.120	1.690	1.255	1.120	.9582	.6953	.6662	1.300	1.187	.5307
1.41	.7355	.3098	2.153	1.707	1.261	1.123	.9557	.6980	.6670	1.307	1.193	.5334
1.42	.7314	.3055	2.186	1.724	1.268	1.126	.9531	.7006	.6677	1.313	1.199	.5361
1.43	.7274	.3012	2.219	1.742	1.274	1.129	.9504	.7032	.6683	1.320	1.205	.5388
1.44	.7235	.2969	2.253	1.759	1.281	1.132	.9476	.7058	.6688	1.326	1.211	.5414
1.45	.7196	.2927	2.286	1.776	1.287	1.135	.9448	.7083	.6692	1.333	1.217	.5441
1.46	.7157	.2886	2.320	1.793	1.294	1.137	.9420	.7108	.6695	1.339	1.222	.5467
1.47	.7120	.2845	2.354	1.811	1.300	1.140	.9390	.7132	.6697	1.346	1.228	.5493
1.48	.7083	.2804	2.389	1.828	1.307	1.143	.9360	.7156	.6698	1.352	1.234	.5519
1.49	.7047	.2764	2.423	1.845	1.314	1.146	.9329	.7179	.6698	1.358	1.240	.5545
1.50	.7011	.2724	2.458	1.862	1.320	1.149	.9298	.7202	.6697	1.365	1.246	.5571
1.51	.6976	.2685	2.493	1.879	1.327	1.152	.9266	.7225	.6694	1.371	1.251	.5596
1.52	.6941	.2646	2.529	1.896	1.334	1.155	.9233	.7247	.6691	1.377	1.257	.5622
1.53	.6907	.2608	2.564	1.913	1.340	1.158	.9200	.7269	.6687	1.383	1.263	.5647
1.54	.6874	.2570	2.600	1.930	1.347	1.161	.9166	.7290	.6682	1.389	1.268	.5672
1.55	.6841	.2533	2.636	1.947	1.354	1.164	.9132	.7311	.6677	1.395	1.274	.5697
1.56	.6809	.2496	2.673	1.964	1.361	1.166	.9097	.7332	.6670	1.402	1.279	.5722
1.57	.6777	.2459	2.709	1.981	1.367	1.169	.9061	.7352	.6662	1.408	1.285	.5746
1.58	.6746	.2423	2.746	1.998	1.374	1.172	.9026	.7372	.6654	1.414	1.290	.5771
1.59	.6715	.2388	2.783	2.015	1.381	1.175	.8989	.7392	.6645	1.419	1.296	.5795
1.60	.6684	.2353	2.820	2.032	1.388	1.178	.8952	.7411	.6635	1.425	1.301	.5819
1.61	.6655	.2318	2.857	2.049	1.395	1.181	.8914	.7430	.6624	1.431	1.307	.5843
1.62	.6625	.2284	2.895	2.065	1.402	1.184	.8877	.7449	.6612	1.437	1.312	.5867
1.63	.6596	.2250	2.933	2.082	1.409	1.187	.8838	.7467	.6600	1.443	1.317	.5891
1.64	.6568	.2217	2.971	2.099	1.416	1.190	.8799	.7485	.6587	1.449	1.322	.5914
1.65	.6540	.2184	3.010	2.115	1.423	1.193	.8760	.7503	.6573	1.454	1.328	.5938
1.66	.6512	.2151	3.048	2.132	1.430	1.196	.8720	.7521	.6558	1.460	1.333	.5961
1.67	.6485	.2119	3.087	2.148	1.437	1.199	.8680	.7538	.6543	1.466	1.338	.5984
1.68	.6458	.2088	3.126	2.165	1.444	1.202	.8640	.7555	.6527	1.471	1.343	.6007
1.69	.6431	.2057	3.165	2.181	1.451	1.205	.8599	.7572	.6511	1.477	1.348	.6030
1.70	.6405	.2026	3.205	2.198	1.458	1.208	.8557	.7588	.6493	1.482	1.353	.6052

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M_0	M_1	p_0/H_0	p_1/p_0	ρ_1/ρ_0	T_1/T_0	a_1/a_0	H_1/H_0	p_1/H_1	p_1/H_0	V_0/a^*	V_0/a_a	$V_0/\hat{V}$
1.70	.6405	.2026	3.205	2.198	1.458	1.208	.8557	.7588	.6493	1.482	1.353	.6052
1.71	.6380	.1996	3.245	2.214	1.466	1.211	.8516	.7604	.6475	1.488	1.358	.6075
1.72	.6355	.1966	3.285	2.230	1.473	1.214	.8474	.7620	.6457	1.493	1.363	.6097
1.73	.6330	.1936	3.325	2.247	1.480	1.217	.8431	.7635	.6438	1.499	1.368	.6119
1.74	.6305	.1907	3.366	2.263	1.487	1.220	.8389	.7651	.6418	1.504	1.373	.6141
1.75	.6281	.1878	3.406	2.279	1.495	1.223	.8346	.7666	.6398	1.510	1.378	.6163
1.76	.6257	.1850	3.447	2.295	1.502	1.226	.8302	.7681	.6377	1.515	1.383	.6185
1.77	.6234	.1822	3.488	2.311	1.509	1.229	.8259	.7696	.6356	1.520	1.388	.6207
1.78	.6210	.1794	3.530	2.327	1.517	1.232	.8215	.7710	.6334	1.526	1.393	.6228
1.79	.6188	.1767	3.571	2.343	1.524	1.235	.8171	.7724	.6311	1.531	1.397	.6249
1.80	.6165	.1740	3.613	2.359	1.532	1.238	.8127	.7738	.6289	1.536	1.402	.6271
1.81	.6143	.1714	3.655	2.375	1.539	1.241	.8082	.7752	.6265	1.541	1.407	.6292
1.82	.6121	.1688	3.698	2.391	1.547	1.244	.8038	.7765	.6242	1.546	1.412	.6313
1.83	.6099	.1662	3.740	2.407	1.554	1.247	.7993	.7779	.6217	1.551	1.416	.6333
1.84	.6078	.1637	3.783	2.422	1.562	1.250	.7948	.7792	.6193	1.556	1.421	.6354
1.85	.6057	.1612	3.826	2.438	1.569	1.253	.7902	.7805	.6168	1.561	1.425	.6375
1.86	.6036	.1587	3.870	2.454	1.577	1.256	.7857	.7818	.6142	1.566	1.430	.6395
1.87	.6016	.1563	3.913	2.469	1.585	1.259	.7811	.7830	.6116	1.571	1.434	.6415
1.88	.5996	.1539	3.957	2.485	1.592	1.262	.7765	.7843	.6090	1.576	1.439	.6435
1.89	.5976	.1516	4.001	2.500	1.600	1.265	.7720	.7855	.6064	1.581	1.443	.6455
1.90	.5956	.1492	4.045	2.516	1.608	1.268	.7674	.7867	.6037	1.586	1.448	.6475
1.91	.5937	.1470	4.089	2.531	1.616	1.271	.7628	.7879	.6009	1.591	1.452	.6495
1.92	.5918	.1447	4.134	2.546	1.624	1.274	.7581	.7890	.5982	1.596	1.457	.6515
1.93	.5899	.1425	4.179	2.562	1.631	1.277	.7535	.7902	.5954	1.600	1.461	.6534
1.94	.5880	.1403	4.224	2.577	1.639	1.280	.7488	.7913	.5926	1.605	1.465	.6553
1.95	.5862	.1381	4.270	2.592	1.647	1.283	.7442	.7925	.5897	1.610	1.470	.6573
1.96	.5844	.1360	4.315	2.607	1.655	1.287	.7395	.7936	.5869	1.615	1.474	.6592
1.97	.5826	.1339	4.361	2.622	1.663	1.290	.7349	.7946	.5840	1.619	1.478	.6611
1.98	.5808	.1318	4.407	2.637	1.671	1.293	.7302	.7957	.5810	1.624	1.482	.6629
1.99	.5791	.1298	4.453	2.652	1.679	1.296	.7255	.7968	.5781	1.628	1.487	.6648
2.00	.5773	.1278	4.500	2.667	1.688	1.299	.7209	.7978	.5751	1.633	1.491	.6667
2.01	.5757	.1258	4.547	2.681	1.696	1.302	.7162	.7988	.5721	1.638	1.495	.6685
2.02	.5740	.1239	4.594	2.696	1.704	1.305	.7115	.7998	.5691	1.642	1.499	.6703
2.03	.5723	.1220	4.641	2.711	1.712	1.308	.7069	.8008	.5661	1.646	1.503	.6722
2.04	.5707	.1201	4.689	2.725	1.720	1.312	.7022	.8018	.5630	1.651	1.507	.6740
2.05	.5691	.1182	4.736	2.740	1.729	1.315	.6975	.8028	.5600	1.655	1.511	.6758
2.06	.5675	.1164	4.784	2.755	1.737	1.318	.6928	.8038	.5569	1.660	1.515	.6776
2.07	.5659	.1146	4.832	2.769	1.745	1.321	.6882	.8047	.5538	1.664	1.519	.6793
2.08	.5643	.1128	4.881	2.783	1.754	1.324	.6835	.8056	.5507	1.668	1.523	.6811
2.09	.5628	.1111	4.929	2.798	1.762	1.327	.6789	.8066	.5475	1.673	1.527	.6828
2.10	.5613	.1094	4.978	2.812	1.770	1.331	.6742	.8075	.5444	1.677	1.531	.6846
2.11	.5598	.1077	5.027	2.826	1.779	1.334	.6696	.8084	.5412	1.681	1.535	.6863
2.12	.5583	.1060	5.077	2.840	1.787	1.337	.6649	.8092	.5381	1.685	1.538	.6880
2.13	.5568	.1043	5.126	2.854	1.796	1.340	.6603	.8101	.5349	1.689	1.542	.6897
2.14	.5554	.1027	5.176	2.868	1.805	1.343	.6557	.8110	.5317	1.694	1.546	.6914
2.15	.5540	.1011	5.226	2.882	1.813	1.347	.6511	.8118	.5285	1.698	1.550	.6931
2.16	.5525	.09956	5.277	2.896	1.822	1.350	.6464	.8127	.5253	1.702	1.554	.6948
2.17	.5511	.09802	5.327	2.910	1.831	1.353	.6419	.8135	.5221	1.706	1.557	.6964
2.18	.5498	.09650	5.378	2.924	1.839	1.356	.6373	.8143	.5189	1.710	1.561	.6981
2.19	.5484	.09500	5.429	2.938	1.848	1.359	.6327	.8151	.5157	1.714	1.565	.6997
2.20	.5471	.09352	5.480	2.951	1.857	1.363	.6281	.8159	.5125	1.718	1.568	.7013
2.21	.5457	.09207	5.531	2.965	1.866	1.366	.6236	.8167	.5093	1.722	1.572	.7029
2.22	.5444	.09064	5.583	2.978	1.875	1.369	.6191	.8175	.5061	1.726	1.575	.7046
2.23	.5431	.08923	5.635	2.992	1.883	1.372	.6145	.8182	.5028	1.730	1.579	.7061
2.24	.5418	.08785	5.687	3.005	1.892	1.376	.6100	.8190	.4996	1.734	1.583	.7077
2.25	.5406	.08648	5.740	3.019	1.901	1.379	.6055	.8197	.4964	1.737	1.586	.7093
2.26	.5393	.08514	5.792	3.032	1.910	1.382	.6011	.8205	.4931	1.741	1.590	.7109
2.27	.5381	.08382	5.845	3.045	1.919	1.385	.5966	.8212	.4899	1.745	1.593	.7124
2.28	.5368	.08251	5.898	3.058	1.929	1.389	.5921	.8219	.4867	1.749	1.596	.7140
2.29	.5356	.08123	5.951	3.071	1.938	1.392	.5877	.8226	.4835	1.753	1.600	.7155
2.30	.5344	.07997	6.005	3.085	1.947	1.395	.5833	.8233	.4802	1.756	1.603	.7170
2.31	.5332	.07873	6.059	3.098	1.956	1.399	.5789	.8240	.4770	1.760	1.607	.7185
2.32	.5321	.07751	6.113	3.110	1.965	1.402	.5745	.8247	.4738	1.764	1.610	.7200
2.33	.5309	.07631	6.167	3.123	1.974	1.405	.5702	.8254	.4706	1.767	1.613	.7215
2.34	.5297	.07512	6.222	3.136	1.984	1.408	.5658	.8260	.4674	1.771	1.617	.7230
2.35	.5286	.07396	6.276	3.149	1.993	1.412	.5615	.8267	.4642	1.775	1.620	.7245
2.36	.5275	.07281	6.331	3.162	2.002	1.415	.5572	.8273	.4610	1.778	1.623	.7259
2.37	.5264	.07168	6.386	3.174	2.012	1.418	.5529	.8280	.4578	1.782	1.626	.7274
2.38	.5253	.07057	6.442	3.187	2.021	1.422	.5486	.8286	.4546	1.785	1.630	.7288
2.39	.5242	.06948	6.497	3.199	2.031	1.425	.5444	.8292	.4514	1.789	1.633	.7302
2.40	.5231	.06840	6.553	3.212	2.040	1.428	.5401	.8299	.4482	1.792	1.636	.7317

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TABLE III.- CONTINUED. NORMAL SHOCK WAVES

M_0	M_1	p_0/H_0	p_1/p_0	ρ_1/ρ_0	T_1/T_0	a_1/a_0	H_1/H_0	p_1/H_1	p_1/H_0	V_0/a^*	V_0/a_a	$V_0/\hat{V}$
2.40	.5231	.06840	6.553	3.212	2.040	1.428	.5401	.8299	.4482	1.792	1.636	.7317
2.41	.5221	.06734	6.609	3.224	2.050	1.432	.5359	.8305	.4451	1.796	1.639	.7331
2.42	.5210	.06630	6.666	3.237	2.059	1.435	.5317	.8311	.4419	1.799	1.642	.7345
2.43	.5200	.06527	6.722	3.249	2.069	1.438	.5276	.8317	.4388	1.802	1.645	.7359
2.44	.5189	.06426	6.779	3.261	2.079	1.442	.5234	.8323	.4356	1.806	1.649	.7372
2.45	.5179	.06327	6.836	3.273	2.088	1.445	.5193	.8328	.4325	1.809	1.652	.7386
2.46	.5169	.06229	6.894	3.285	2.098	1.449	.5152	.8334	.4294	1.813	1.655	.7400
2.47	.5159	.06133	6.951	3.298	2.108	1.452	.5111	.8340	.4263	1.816	1.658	.7413
2.48	.5149	.06038	7.009	3.310	2.118	1.455	.5071	.8346	.4232	1.819	1.661	.7427
2.49	.5140	.05945	7.067	3.321	2.128	1.459	.5030	.8351	.4201	1.822	1.664	.7440
2.50	.5130	.05853	7.125	3.333	2.138	1.462	.4990	.8357	.4170	1.826	1.667	.7454
2.51	.5120	.05762	7.183	3.345	2.147	1.465	.4950	.8362	.4139	1.829	1.670	.7467
2.52	.5111	.05674	7.242	3.357	2.157	1.469	.4911	.8367	.4109	1.832	1.673	.7480
2.53	.5102	.05586	7.301	3.369	2.167	1.472	.4871	.8373	.4078	1.835	1.675	.7493
2.54	.5092	.05500	7.360	3.380	2.177	1.476	.4832	.8378	.4048	1.839	1.678	.7506
2.55	.5083	.05415	7.420	3.392	2.187	1.479	.4793	.8383	.4018	1.842	1.681	.7519
2.56	.5074	.05332	7.479	3.403	2.198	1.482	.4754	.8388	.3988	1.845	1.684	.7531
2.57	.5065	.05250	7.539	3.415	2.208	1.486	.4715	.8393	.3958	1.848	1.687	.7544
2.58	.5056	.05169	7.599	3.426	2.218	1.489	.4677	.8399	.3928	1.851	1.690	.7557
2.59	.5047	.05090	7.659	3.438	2.228	1.493	.4639	.8403	.3898	1.854	1.693	.7569
2.60	.5039	.05012	7.720	3.449	2.238	1.496	.4601	.8408	.3869	1.857	1.695	.7582
2.61	.5030	.04935	7.781	3.460	2.249	1.500	.4564	.8413	.3839	1.860	1.698	.7594
2.62	.5022	.04859	7.842	3.471	2.259	1.503	.4526	.8418	.3810	1.863	1.701	.7606
2.63	.5013	.04784	7.903	3.483	2.269	1.506	.4489	.8423	.3781	1.866	1.704	.7619
2.64	.5005	.04711	7.965	3.494	2.280	1.510	.4452	.8428	.3752	1.869	1.706	.7631
2.65	.4996	.04639	8.026	3.505	2.290	1.513	.4416	.8432	.3723	1.872	1.709	.7643
2.66	.4988	.04568	8.088	3.516	2.301	1.517	.4379	.8437	.3695	1.875	1.712	.7655
2.67	.4980	.04498	8.150	3.527	2.311	1.520	.4343	.8441	.3666	1.878	1.714	.7667
2.68	.4972	.04429	8.213	3.537	2.322	1.524	.4307	.8446	.3638	1.881	1.717	.7678
2.69	.4964	.04362	8.275	3.548	2.332	1.527	.4271	.8450	.3609	1.884	1.720	.7690
2.70	.4956	.04295	8.338	3.559	2.343	1.531	.4236	.8455	.3581	1.887	1.722	.7702
2.71	.4949	.04229	8.401	3.570	2.354	1.534	.4201	.8459	.3553	1.889	1.725	.7713
2.72	.4941	.04165	8.465	3.580	2.364	1.538	.4166	.8463	.3526	1.892	1.727	.7725
2.73	.4933	.04102	8.528	3.591	2.375	1.541	.4131	.8468	.3498	1.895	1.730	.7736
2.74	.4926	.04039	8.592	3.601	2.386	1.545	.4097	.8472	.3470	1.898	1.732	.7748
2.75	.4918	.03978	8.656	3.612	2.397	1.548	.4062	.8476	.3443	1.901	1.735	.7759
2.76	.4911	.03917	8.721	3.622	2.407	1.552	.4028	.8480	.3416	1.903	1.737	.7770
2.77	.4903	.03858	8.785	3.633	2.418	1.555	.3994	.8484	.3389	1.906	1.740	.7781
2.78	.4896	.03799	8.850	3.643	2.429	1.559	.3961	.8488	.3362	1.909	1.742	.7792
2.79	.4889	.03742	8.915	3.653	2.440	1.562	.3928	.8492	.3335	1.911	1.745	.7803
2.80	.4882	.03685	8.980	3.664	2.451	1.566	.3895	.8496	.3309	1.914	1.747	.7814
2.81	.4875	.03630	9.045	3.674	2.462	1.569	.3862	.8500	.3283	1.917	1.750	.7825
2.82	.4868	.03574	9.111	3.684	2.473	1.573	.3829	.8504	.3256	1.919	1.752	.7836
2.83	.4861	.03520	9.177	3.694	2.484	1.576	.3797	.8508	.3230	1.922	1.754	.7846
2.84	.4854	.03467	9.243	3.704	2.496	1.580	.3765	.8512	.3205	1.925	1.757	.7857
2.85	.4847	.03415	9.310	3.714	2.507	1.583	.3733	.8515	.3179	1.927	1.759	.7868
2.86	.4840	.03363	9.376	3.724	2.518	1.587	.3701	.8519	.3153	1.930	1.762	.7878
2.87	.4833	.03312	9.443	3.734	2.529	1.590	.3670	.8523	.3128	1.932	1.764	.7888
2.88	.4827	.03263	9.510	3.743	2.540	1.594	.3639	.8527	.3103	1.935	1.766	.7899
2.89	.4820	.03213	9.577	3.753	2.552	1.597	.3608	.8530	.3078	1.937	1.769	.7909
2.90	.4814	.03165	9.645	3.763	2.563	1.601	.3577	.8534	.3053	1.940	1.771	.7919
2.91	.4807	.03118	9.713	3.773	2.575	1.605	.3547	.8537	.3028	1.942	1.773	.7929
2.92	.4801	.03071	9.781	3.782	2.586	1.608	.3517	.8541	.3004	1.945	1.775	.7939
2.93	.4795	.03025	9.849	3.792	2.598	1.612	.3487	.8544	.2979	1.947	1.778	.7949
2.94	.4788	.02980	9.918	3.801	2.609	1.615	.3457	.8548	.2955	1.950	1.780	.7959
2.95	.4782	.02935	9.986	3.811	2.621	1.619	.3428	.8551	.2931	1.952	1.782	.7969
2.96	.4776	.02891	10.06	3.820	2.632	1.622	.3398	.8554	.2907	1.954	1.784	.7979
2.97	.4770	.02848	10.12	3.829	2.644	1.626	.3369	.8558	.2883	1.957	1.786	.7989
2.98	.4764	.02805	10.19	3.839	2.656	1.630	.3340	.8561	.2860	1.959	1.789	.7999
2.99	.4758	.02764	10.26	3.848	2.667	1.633	.3312	.8564	.2836	1.962	1.791	.8008
3.00	.4752	.02722	10.33	3.857	2.679	1.637	.3283	.8568	.2813	1.964	1.793	.8018
3.01	.4746	.02682	10.40	3.866	2.691	1.640	.3255	.8571	.2790	1.966	1.795	.8027
3.02	.4740	.02642	10.47	3.875	2.703	1.644	.3227	.8574	.2767	1.969	1.797	.8037
3.03	.4734	.02603	10.54	3.884	2.714	1.648	.3200	.8577	.2744	1.971	1.799	.8046
3.04	.4729	.02564	10.62	3.893	2.726	1.651	.3172	.8580	.2722	1.973	1.801	.8056
3.05	.4723	.02526	10.69	3.902	2.738	1.655	.3145	.8583	.2699	1.975	1.803	.8065
3.06	.4717	.02489	10.76	3.911	2.750	1.658	.3118	.8587	.2677	1.978	1.805	.8074
3.07	.4712	.02452	10.83	3.920	2.762	1.662	.3091	.8589	.2655	1.980	1.807	.8083
3.08	.4706	.02416	10.90	3.929	2.774	1.666	.3065	.8592	.2633	1.982	1.809	.8092
3.09	.4701	.02380	10.97	3.938	2.786	1.669	.3038	.8595	.2611	1.984	1.812	.8101
3.10	.4695	.02345	11.05	3.947	2.799	1.673	.3012	.8598	.2590	1.987	1.814	.8110

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TABLE III.- CONTINUED. NORMAL SHOCK WAVES

M_0	M_1	p_0/H_0	p_1/p_0	ρ_1/ρ_0	T_1/T_0	a_1/a_0	H_1/H_0	p_1/H_1	p_1/H_0	V_0/a^*	V_0/a_a	$V_0/\hat{V}$
3.10	.4695	.02345	11.05	3.947	2.799	1.673	.3012	.8598	.2590	1.987	1.814	.8110
3.11	.4690	.02310	11.12	3.955	2.811	1.677	.2986	.8601	.2568	1.989	1.816	.8119
3.12	.4685	.02276	11.19	3.964	2.823	1.680	.2960	.8604	.2547	1.991	1.818	.8128
3.13	.4679	.02243	11.26	3.973	2.835	1.684	.2935	.8607	.2526	1.993	1.819	.8137
3.14	.4674	.02210	11.34	3.981	2.848	1.687	.2910	.8610	.2505	1.995	1.821	.8146
3.15	.4669	.02177	11.41	3.990	2.860	1.691	.2885	.8613	.2484	1.997	1.823	.8154
3.16	.4664	.02146	11.48	3.998	2.872	1.695	.2860	.8615	.2464	2.000	1.825	.8163
3.17	.4659	.02114	11.56	4.006	2.885	1.698	.2835	.8618	.2443	2.002	1.827	.8172
3.18	.4654	.02083	11.63	4.015	2.897	1.702	.2811	.8621	.2423	2.004	1.829	.8180
3.19	.4648	.02053	11.71	4.023	2.909	1.706	.2786	.8624	.2403	2.006	1.831	.8189
3.20	.4643	.02023	11.78	4.031	2.922	1.709	.2762	.8626	.2383	2.008	1.833	.8197
3.21	.4639	.01993	11.85	4.040	2.935	1.713	.2738	.8629	.2363	2.010	1.835	.8205
3.22	.4634	.01964	11.93	4.048	2.947	1.717	.2715	.8632	.2343	2.012	1.837	.8214
3.23	.4629	.01936	12.01	4.056	2.960	1.720	.2691	.8634	.2324	2.014	1.839	.8222
3.24	.4624	.01908	12.08	4.064	2.972	1.724	.2668	.8637	.2304	2.016	1.840	.8230
3.25	.4619	.01880	12.16	4.072	2.985	1.728	.2645	.8639	.2285	2.018	1.842	.8238
3.26	.4614	.01853	12.23	4.080	2.998	1.731	.2622	.8642	.2266	2.020	1.844	.8247
3.27	.4610	.01826	12.31	4.088	3.011	1.735	.2600	.8644	.2247	2.022	1.846	.8255
3.28	.4605	.01799	12.38	4.096	3.023	1.739	.2577	.8647	.2228	2.024	1.848	.8263
3.29	.4600	.01773	12.46	4.104	3.036	1.742	.2555	.8649	.2210	2.026	1.849	.8271
3.30	.4596	.01748	12.54	4.112	3.049	1.746	.2533	.8652	.2191	2.028	1.851	.8279
3.31	.4591	.01722	12.62	4.120	3.062	1.750	.2511	.8654	.2173	2.030	1.853	.8286
3.32	.4587	.01698	12.69	4.128	3.075	1.754	.2489	.8657	.2155	2.032	1.855	.8294
3.33	.4582	.01673	12.77	4.135	3.088	1.757	.2468	.8659	.2137	2.034	1.856	.8302
3.34	.4578	.01649	12.85	4.143	3.101	1.761	.2446	.8661	.2119	2.035	1.858	.8310
3.35	.4573	.01625	12.93	4.151	3.114	1.765	.2425	.8664	.2101	2.037	1.860	.8317
3.36	.4569	.01602	13.00	4.158	3.127	1.768	.2404	.8666	.2084	2.039	1.862	.8325
3.37	.4565	.01579	13.08	4.166	3.141	1.772	.2383	.8668	.2066	2.041	1.863	.8333
3.38	.4560	.01557	13.16	4.173	3.154	1.776	.2363	.8671	.2049	2.043	1.865	.8340
3.39	.4556	.01534	13.24	4.181	3.167	1.780	.2342	.8673	.2032	2.045	1.867	.8348
3.40	.4552	.01512	13.32	4.188	3.180	1.783	.2322	.8675	.2015	2.047	1.868	.8355
3.41	.4548	.01491	13.40	4.196	3.194	1.787	.2302	.8677	.1998	2.048	1.870	.8362
3.42	.4544	.01470	13.48	4.203	3.207	1.791	.2282	.8680	.1981	2.050	1.872	.8370
3.43	.4540	.01449	13.56	4.211	3.220	1.795	.2263	.8682	.1964	2.052	1.873	.8377
3.44	.4535	.01428	13.64	4.218	3.234	1.798	.2243	.8684	.1948	2.054	1.875	.8384
3.45	.4531	.01408	13.72	4.225	3.247	1.802	.2224	.8686	.1932	2.056	1.876	.8392
3.46	.4527	.01388	13.80	4.232	3.261	1.806	.2205	.8688	.1915	2.057	1.878	.8399
3.47	.4523	.01368	13.88	4.240	3.274	1.809	.2186	.8690	.1899	2.059	1.880	.8406
3.48	.4519	.01349	13.96	4.247	3.288	1.813	.2167	.8692	.1883	2.061	1.881	.8413
3.49	.4515	.01330	14.04	4.254	3.301	1.817	.2148	.8695	.1868	2.062	1.883	.8420
3.5	.4512	.01311	14.13	4.261	3.315	1.821	.2129	.8697	.1852	2.064	1.884	.8427
3.6	.4474	.01138	14.95	4.330	3.454	1.858	.1953	.8716	.1702	2.081	1.899	.8495
3.7	.4439	9.903×10^{-3}	15.80	4.395	3.596	1.896	.1792	.8734	.1565	2.096	1.914	.8558
3.8	.4407	8.629×10^{-3}	16.68	4.457	3.743	1.935	.1645	.8751	.1439	2.111	1.927	.8619
3.9	.4377	7.532×10^{-3}	17.58	4.516	3.893	1.973	.1510	.8767	.1324	2.125	1.940	.8675
4.0	.4350	6.586×10^{-3}	18.50	4.571	4.047	2.012	.1388	.8781	.1218	2.138	1.952	.8729
4.1	.4324	5.769×10^{-3}	19.45	4.624	4.205	2.051	.1276	.8794	.1122	2.150	1.963	.8779
4.2	.4299	5.062×10^{-3}	20.41	4.675	4.367	2.090	.1173	.8807	.1033	2.162	1.974	.8827
4.3	.4277	4.449×10^{-3}	21.41	4.723	4.532	2.129	.1080	.8818	.09524	2.173	1.984	.8872
4.4	.4255	3.918×10^{-3}	22.42	4.768	4.702	2.168	.09948	.8829	.08783	2.184	1.993	.8915
4.5	.4236	3.455×10^{-3}	23.46	4.812	4.875	2.208	.09170	.8839	.08105	2.194	2.002	.8955
4.6	.4217	3.053×10^{-3}	24.52	4.853	5.052	2.248	.08459	.8849	.07485	2.203	2.011	.8994
4.7	.4199	2.701×10^{-3}	25.61	4.893	5.233	2.288	.07809	.8858	.06917	2.212	2.019	.9030
4.8	.4183	2.394×10^{-3}	26.71	4.930	5.418	2.328	.07214	.8866	.06396	2.220	2.027	.9065
4.9	.4167	2.126×10^{-3}	27.85	4.966	5.607	2.368	.06670	.8874	.05919	2.228	2.034	.9098
5.0	.4152	1.890×10^{-3}	29.00	5.000	5.800	2.408	.06172	.8881	.05481	2.236	2.041	.9129
6.0	.4042	6.334×10^{-4}	41.83	5.268	7.941	2.818	.02965	.8936	.02650	2.295	2.095	.9370
7.0	.3974	2.416×10^{-4}	57.00	5.444	10.47	3.236	.01535	.8969	.01377	2.333	2.130	.9526

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TABLE III.– CONCLUDED. NORMAL SHOCK WAVES

M_o	M_1	p_o/H_o	p_1/p_o	ρ_1/ρ_o	T_1/T_o	a_1/a_o	H_1/H_o	p_1/H_1	p_1/H_o	V_o/a^*	V_o/a_a	$V_o/\hat{V}$
7	.3974	2.416×10^{-4}	57.00	5.444	10.47	3.236	.01535	.8969	.01377	2.333	2.130	.9526
8	.3929	1.024×10^{-4}	74.50	5.565	13.39	3.659	8.488×10^{-3}	.8990	7.631×10^{-3}	2.359	2.154	.9631
9	.3898	4.739×10^{-5}	94.33	5.651	16.69	4.086	4.964×10^{-3}	.9005	4.470×10^{-3}	2.377	2.170	.9705
10	.3876	2.356×10^{-5}	116.5	5.714	20.39	4.515	3.045×10^{-3}	.9016	2.745×10^{-3}	2.390	2.182	.9759
15	.3823	1.515×10^{-6}	262.3	5.870	44.69	6.685	4.395×10^{-4}	.9041	3.974×10^{-4}	2.423	2.212	.9891
20	.3804	2.091×10^{-7}	466.5	5.926	78.72	8.873	1.078×10^{-4}	.9050	9.753×10^{-5}	2.434	2.222	.9938
100	.3781	2.790×10^{-12}	11,666.5	5.997	1945.4	44.11	3.593×10^{-8}	.9061	3.255×10^{-8}	2.449	2.236	.9998
∞	.3780	0	∞	6	∞	∞	0	.9061	0	2.449	2.236	1.0000

Definition of Symbols for Table III

M_o	Mach number upstream of normal shock wave
M_1	Mach number downstream of normal shock wave
p_o/H_o	ratio of static pressure to total pressure upstream of shock wave
p_1/p_o	static pressure ratio across shock wave
ρ_1/ρ_o	density ratio across shock wave
T_1/T_o	temperature ratio across shock wave
a_1/a_o	local speed of sound ratio across shock wave
H_1/H_o	ratio of total pressure downstream of shock wave to total pressure upstream
p_1/H_1	ratio of static pressure to total pressure downstream of shock wave
p_1/H_o	ratio of static pressure downstream to total pressure upstream of wave
V_o/a^*	ratio of velocity (corresponding to M_o) to the speed of sound where $V = a$
V_o/a_a	ratio of velocity (corresponding to M_o) to the speed of sound where $V = o$
$V_o/\hat{V}$	ratio of velocity (corresponding to M_o) to the velocity where $p = \rho = T = o$

TABLE IV.- MACH NUMBER FUNCTIONS FOR USE WITH SMALL-PERTURBATION AIRFOIL-SECTION THEORY

M_0	C_1	C_2	M_0	C_1	C_2	M_0	C_1	C_2
1.02	9.950	746.293	2.02	1.140	1.456	3.02	.702	1.268
1.04	7.001	186.333	2.04	1.125	1.447	3.04	.697	1.266
1.06	5.689	82.987	2.06	1.110	1.437	3.06	.692	1.265
1.08	4.903	46.943	2.08	1.097	1.429	3.08	.687	1.264
1.10	4.364	30.315	2.10	1.083	1.420	3.10	.682	1.263
1.12	3.965	21.313	2.12	1.070	1.413	3.12	.677	1.262
1.14	3.654	15.905	2.14	1.057	1.405	3.14	.672	1.260
1.16	3.402	12.404	2.16	1.045	1.398	3.16	.667	1.259
1.18	3.193	10.013	2.18	1.032	1.392	3.18	.663	1.258
1.20	3.015	8.307	2.20	1.021	1.386	3.20	.658	1.257
1.22	2.862	7.050	2.22	1.009	1.380	3.22	.653	1.256
1.24	2.728	6.096	2.24	.998	1.374	3.24	.649	1.256
1.26	2.609	5.356	2.26	.987	1.369	3.26	.645	1.255
1.28	2.503	4.771	2.28	.976	1.363	3.28	.640	1.254
1.30	2.408	4.300	2.30	.966	1.358	3.30	.636	1.253
1.32	2.321	3.916	2.32	.955	1.354	3.32	.632	1.252
1.34	2.242	3.598	2.34	.945	1.349	3.34	.628	1.251
1.36	2.170	3.333	2.36	.936	1.345	3.36	.623	1.250
1.38	2.103	3.109	2.38	.926	1.341	3.38	.619	1.249
1.40	2.041	2.919	2.40	.917	1.337	3.40	.615	1.249
1.42	1.984	2.755	2.42	.908	1.333	3.42	.612	1.248
1.44	1.930	2.614	2.44	.899	1.330	3.44	.608	1.247
1.46	1.880	2.491	2.46	.890	1.326	3.46	.604	1.246
1.48	1.833	2.383	2.48	.881	1.323	3.48	.600	1.246
1.50	1.789	2.288	2.50	.873	1.320	3.50	.596	1.245
1.52	1.747	2.204	2.52	.865	1.317	3.60	.578	1.242
1.54	1.708	2.129	2.54	.857	1.314	3.70	.561	1.239
1.56	1.670	2.063	2.56	.849	1.311	3.80	.546	1.236
1.58	1.635	2.003	2.58	.841	1.308	3.90	.531	1.234
1.60	1.601	1.949	2.60	.833	1.306	4.00	.516	1.232
1.62	1.569	1.901	2.62	.826	1.303	4.10	.503	1.230
1.64	1.539	1.858	2.64	.819	1.301	4.20	.490	1.228
1.66	1.509	1.817	2.66	.811	1.298	4.30	.478	1.227
1.68	1.481	1.781	2.68	.804	1.296	4.40	.467	1.225
1.70	1.455	1.748	2.70	.797	1.294	4.50	.456	1.224
1.72	1.429	1.717	2.72	.791	1.292	4.60	.445	1.223
1.74	1.405	1.689	2.74	.784	1.290	4.70	.436	1.222
1.76	1.381	1.663	2.76	.777	1.288	4.80	.426	1.221
1.78	1.358	1.640	2.78	.771	1.286	4.90	.417	1.220
1.80	1.336	1.618	2.80	.765	1.284	5.00	.408	1.219
1.82	1.315	1.597	2.82	.759	1.282	6.00	.338	1.212
1.84	1.295	1.579	2.84	.752	1.281	7.00	.289	1.209
1.86	1.275	1.561	2.86	.746	1.279	8.00	.252	1.207
1.88	1.256	1.545	2.88	.741	1.277	9.00	.224	1.205
1.90	1.238	1.529	2.90	.735	1.276	10.00	.201	1.204
1.92	1.220	1.515	2.92	.729	1.274	15	.134	1.202
1.94	1.203	1.502	2.94	.723	1.273	20	.100	1.201
1.96	1.186	1.489	2.96	.718	1.271	100	.020	1.200
1.98	1.170	1.478	2.98	.712	1.270			
2.00	1.155	1.467	3.00	.707	1.269			

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TABLE V.- PROPERTIES OF THE STANDARD ATMOSPHERE

h (ft)	t (°F)	t (°C)	a/a_{SL}	a (ft/sec)	a (mph)	p/p_{SL}	p (lb/sq ft)	p milli-bars	$\sigma = \rho/\rho_{SL}$	ρ (slug/cu ft)	$\sigma^{\frac{1}{2}}$	$\mu \times 10^7$ (slug/ft sec)	$\nu \times 10^4$ (ft²/sec)	q/M^2 (lb/ft²)
0	59.00	15.00	1.0000	1117	761.6	1.0000	2116.2	1013.2	1.0000	0.002378	1.0000	3.719	1.564	1481.3
1,000	57.44	13.02	.9966	1113	759.0	.9644	2040.9	977.1	.9711	.002310	.9854	3.699	1.602	1428.6
2,000	51.87	11.04	.9931	1109	756.3	.9298	1967.7	942.1	.9428	.002242	.9710	3.679	1.641	1377.4
3,000	48.31	9.06	.9896	1105	753.7	.8963	1896.7	908.1	.9152	.002177	.9566	3.659	1.681	1327.7
4,000	44.74	7.08	.9862	1102	751.0	.8637	1827.7	875.1	.8881	.002112	.9424	3.639	1.723	1279.4
5,000	41.18	5.10	.9827	1098	748.4	.8321	1760.8	843.0	.8617	.002049	.9283	3.618	1.766	1232.6
6,000	37.62	3.12	.9792	1094	745.7	.8014	1696.0	812.0	.8359	.001988	.9143	3.598	1.810	1187.2
7,000	34.05	1.14	.9756	1090	743.0	.7717	1633.0	781.8	.8107	.001928	.9004	3.577	1.855	1143.1
8,000	30.49	- 0.84	.9721	1086	740.4	.7428	1571.9	752.6	.7860	.001869	.8866	3.557	1.903	1100.3
9,000	26.92	- 2.82	.9686	1082	737.7	.7148	1512.8	724.3	.7620	.001812	.8729	3.536	1.951	1059.0
10,000	23.36	- 4.80	.9650	1078	734.9	.6877	1455.4	696.8	.7385	.001756	.8594	3.515	2.002	1018.8
11,000	19.80	- 6.78	.9614	1074	732.2	.6614	1399.8	670.2	.7156	.001702	.8459	3.495	2.054	979.9
12,000	16.23	- 8.76	.9579	1070	729.5	.6360	1345.9	644.4	.6932	.001649	.8326	3.474	2.107	942.1
13,000	12.67	-10.74	.9543	1066	726.8	.6113	1293.7	619.4	.6714	.001597	.8194	3.453	2.163	905.6
14,000	9.10	-12.72	.9507	1062	724.0	.5875	1243.2	595.2	.6500	.001546	.8063	3.432	2.220	870.2
15,000	5.54	-14.70	.9470	1058	721.2	.5644	1194.3	571.8	.6292	.001497	.7933	3.411	2.280	836.0
16,000	1.98	-16.68	.9434	1054	718.5	.5420	1147.0	549.1	.6090	.001448	.7804	3.390	2.341	802.9
17,000	- 1.59	-18.66	.9397	1050	715.7	.5203	1101.1	527.2	.5892	.001401	.7676	3.369	2.404	770.8
18,000	- 5.15	-20.64	.9361	1046	712.9	.4994	1056.9	506.0	.5699	.001355	.7549	3.347	2.470	739.8
19,000	- 8.72	-22.62	.9324	1041	710.1	.4792	1014.0	485.5	.5511	.001311	.7424	3.326	2.538	709.8
20,000	-12.28	-24.60	.9287	1037	707.3	.4596	972.6	465.6	.5328	.001267	.7299	3.305	2.608	680.8
21,000	-15.84	-26.58	.9250	1033	704.5	.4406	932.5	446.4	.5150	.001225	.7176	3.283	2.681	652.8
22,000	-19.41	-28.56	.9213	1029	701.6	.4223	893.8	427.9	.4976	.001183	.7054	3.262	2.757	625.7
23,000	-22.97	-30.54	.9175	1025	698.8	.4047	856.4	410.0	.4807	.001143	.6933	3.240	2.834	599.5
24,000	-26.54	-32.52	.9138	1021	695.9	.3876	820.3	392.7	.4642	.001104	.6813	3.218	2.915	574.2
25,000	-30.10	-34.50	.9100	1017	693.1	.3711	785.3	376.0	.4481	.001066	.6694	3.196	2.999	549.7
26,000	-33.66	-36.48	.9062	1012	690.2	.3552	751.7	359.9	.4325	.001029	.6576	3.174	3.087	526.2
27,000	-37.23	-38.46	.9024	1008	687.3	.3399	719.2	344.3	.4173	.000993	.6460	3.153	3.177	503.4
28,000	-40.79	-40.44	.8986	1004	684.4	.3251	687.9	329.3	.4025	.000957	.6345	3.130	3.270	481.5
29,000	-44.36	-42.42	.8948	999	681.5	.3108	657.6	314.9	.3882	.000923	.6230	3.108	3.367	460.3
30,000	-47.92	-44.40	.8909	995	678.5	.2970	628.5	300.9	.3741	.000890	.6116	3.086	3.469	440.0
31,000	-51.48	-46.38	.8871	991	675.6	.2837	600.4	287.5	.3606	.000858	.6005	3.064	3.573	420.3
32,000	-55.05	-48.36	.8832	987	672.6	.2709	573.3	274.5	.3473	.000826	.5894	3.041	3.682	401.3
33,000	-58.61	-50.34	.8793	982	669.7	.2586	547.3	262.0	.3345	.000796	.5784	3.019	3.795	383.1
34,000	-62.18	-52.32	.8754	978	666.7	.2467	522.2	250.0	.3220	.000766	.5675	2.997	3.913	365.5
35,000	-65.74	-54.30	.8714	973	663.7	.2353	498.0	238.4	.3099	.000737	.5567	2.974	4.036	348.6
35,332	-67.6	-55.33	.8693	971	662.1	.2314	489.8	234.5	.3058	.000727	.5530	2.961	4.073	342.9
36,000	-67.6	-56.28	.8693	971	662.1	.2244	474.8	227.3	.2981	.000709	.5460	2.951	4.176	332.4
37,000	-67.6	-55.33	.8693	971	662.1	.2138	452.5	216.7	.2845	.0006766	.5334	2.961	4.376	316.8
38,000	-67.6	-55.33	.8693	971	662.1	.2038	431.2	206.5	.2711	.0006448	.5207	2.961	4.592	301.8
39,000	-67.6	-55.33	.8693	971	662.1	.1942	411.0	196.8	.2584	.0006145	.5083	2.961	4.819	287.7
40,000	-67.6	-55.33	.8693	971	662.1	.1851	391.8	187.6	.2463	.0005857	.4963	2.961	5.055	274.3
41,000	-67.6	-55.33	.8693	971	662.1	.1764	373.4	178.8	.2347	.0005582	.4845	2.961	5.305	261.4
42,000	-67.6	-55.33	.8693	971	662.1	.1681	355.8	170.4	.2237	.0005320	.4730	2.961	5.566	249.1
43,000	-67.6	-55.33	.8693	971	662.1	.1603	339.1	162.4	.2132	.0005071	.4617	2.961	5.839	237.4
44,000	-67.6	-55.33	.8693	971	662.1	.1527	323.2	154.8	.2032	.0004833	.4508	2.961	6.127	226.2
45,000	-67.6	-55.33	.8693	971	662.1	.1456	308.0	147.5	.1936	.0004605	.4400	2.961	6.430	215.6
46,000	-67.6	-55.33	.8693	971	662.1	.1387	293.6	140.6	.1846	.0004390	.4296	2.961	6.745	205.5
47,000	-67.6	-55.33	.8693	971	662.1	.1322	279.8	134.0	.1759	.0004184	.4194	2.961	7.077	195.9
48,000	-67.6	-55.33	.8693	971	662.1	.1260	266.6	127.7	.1676	.0003987	.4094	2.961	7.427	186.6
49,000	-67.6	-55.33	.8693	971	662.1	.1201	254.1	121.7	.1598	.0003800	.3997	2.961	7.792	177.9
50,000	-67.6	-55.33	.8693	971	662.1	.1145	242.2	116.0	.1523	.0003622	.3902	2.961	8.175	169.5
60,000	-67.6	-55.33	.8693	971	662.1	.0713	150.9	72.2	.0942	.0002240	.3069	2.961	13.219	105.6
70,000	-67.6	-55.33	.8693	971	662.1	.0442	93.5	44.8	.0584	.0001389	.2417	2.961	21.317	65.5
80,000	-67.6	-55.33	.8693	971	662.1	.0274	58.0	27.8	.0362	.0000861	.1903	2.961	34.390	40.6
90,000	-67.6	-55.33	.8693	971	662.1	.0170	36.0	17.2	.0225	.0000535	.1500	2.961	55.346	25.2
100,000	-67.6	-55.33	.8693	971	662.1	.0106	22.4	10.7	.0140	.0000331	.1183	2.961	89.456	15.7
104,987	-67.6	-55.33	.8693	971	662.1	.00831	17.59	8.42	.0110	.0000261	.1048	2.961	113.4	12.31
110,000	-47.4	-44.11	.8917	996	679.0	.00658	13.92	6.66	.00827	.0000197	.09093	3.090	157.2	9.744
120,000	- 7.2	-21.78	.9346	1043	711.1	.00426	9.026	4.32	.00488	.0000116	.06988	3.339	287.6	6.318
130,000	33.0	.56	.9749	1089	742.5	.00287	6.071	2.91	.00302	.00000717	.05493	3.579	498.9	4.250
140,000	73.3	22.94	1.0143	1132	771.8	.00199	4.213	2.02	.00193	.00000460	.04399	3.809	827.9	2.949
150,000	113.5	45.28	1.0510	1174	800.5	.00142	3.003	1.45	.00128	.00000305	.03582	4.032	1322	2.102
160,000	153.7	67.61	1.0877	1215	828.4	.00103	2.190	1.05	.00087	.00000208	.02957	4.247	2043	1.533
164,042	170.0	76.67	1.1021	1231	839.3	.000916	1.938	.928	.00075	.00000179	.02746	4.332	2417	1.357
170,000	170.0	76.67	1.1021	1231	839.3	.000767	1.624	.777	.00063	.00000150	.02513	4.332	2886	1.137
180,000	170.0	76.67	1.1021	1231	839.3	.000570	1.206	.577	.00047	.00000111	.02165	4.332	3885	.8442
190,000	170.0	76.67	1.1021	1231	839.3	.000423	.8956	.429	.00035	.00000083	.01866	4.332	5232	.6269
196,850	170.0	76.67	1.1021	1231	839.3	.000345	.7305	.350	.00028	.00000068	.01685	4.332	6412	.5114
200,000	159.4	70.78	1.0931	1220	831.8	.000314	.6645	.318	.00026	.00000062	.01621	4.277	6844	.4652
210,000	125.9	52.17	1.0627	1187	809.3	.000230	.4869	.233	.00020	.00000048	.01427	4.099	8467	.3408
220,000	92.4	33.56	1.0322	1152	785.5	.000166	.3504	.168	.00015	.00000037	.01247	3.916	10600	.2453
230,000	58.9	14.94	1.0000	1117	761.6	.000117	.2470	.118	.00012	.00000028	.01080	3.727	13400	.1729
240,000	25.3	-3.72	.9669	1080	736.4	.000080	.1699	.081	.00009	.00000020	.00926	3.533	17330	.1189
250,000	- 8.2	-22.33	.9329	1042	710.5	.000054	.1139	.054	.00006	.00000015	.00786	3.333	22700	.0797
255,905	-28.0	-33.33	.9123	1019	694.8	.000042	.0886	.042	.00005	.00000012	.00709	3.212	26880	.0621
260,000	-28.0	-33.33	.9123	1019	694.8	.000035	.0742	.035	.00004	.00000010	.00649	3.212	32090	.0520

List of symbols:
- h = height
- p = pressure
- μ = coefficient of viscosity
- t = temperature
- ρ = density
- ν = coefficient of kinematic viscosity
- a = velocity of sound
- σ = density relative to sea level

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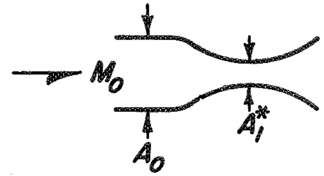
The subscript SL refers to sea level (h = 0)

$$\frac{A_0}{A_1^*}=\frac{\left(\frac{\gamma+1}{\gamma-1}M_0\right)^{\frac{\gamma+1}{\gamma-1}}\left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}{\left(1+\frac{\gamma-1}{2}\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}\left(\frac{2}{\gamma-1}+M_0^2\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{2\gamma}{\gamma-1}M_0^2-1\right)^{\frac{1}{\gamma-1}}}=\frac{27{,}000\,M_0^6\left(1+0.2M_0^2\right)^3}{\left(5+M_0^2\right)^{3.5}\left(7M_0^2-1\right)^{2.5}}$$



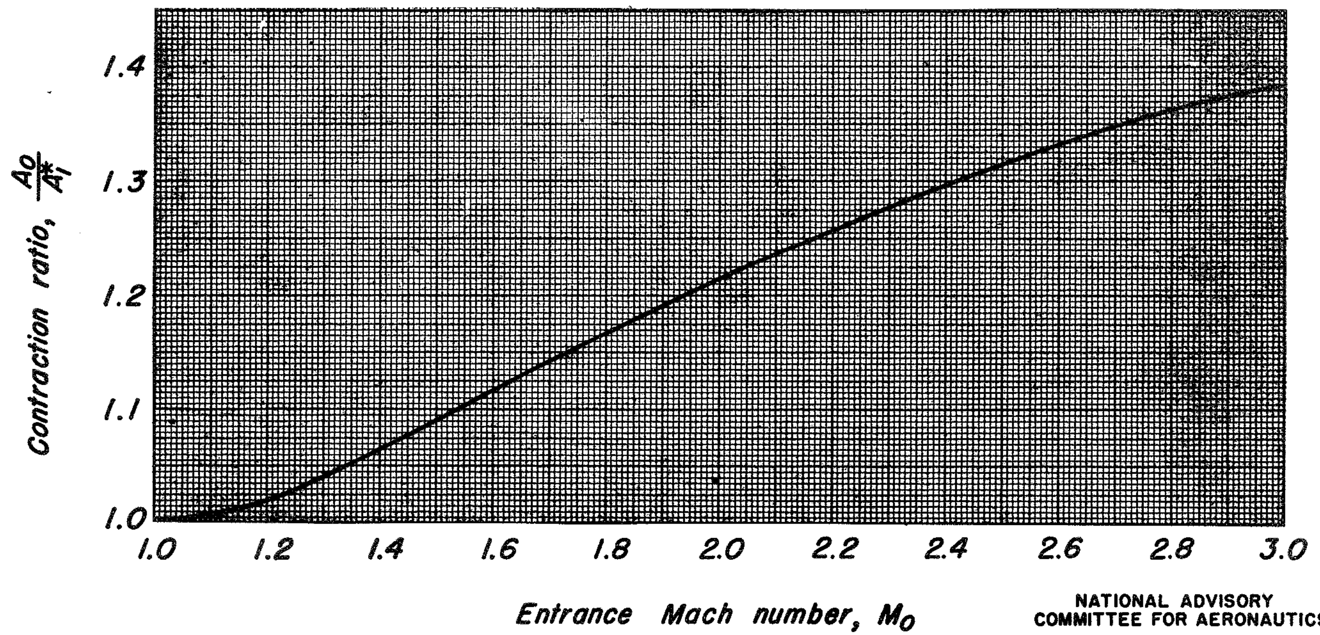


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Figure 1.– Maximum theoretical contraction ratio that permits start of supersonic flow in diffuser entrance.

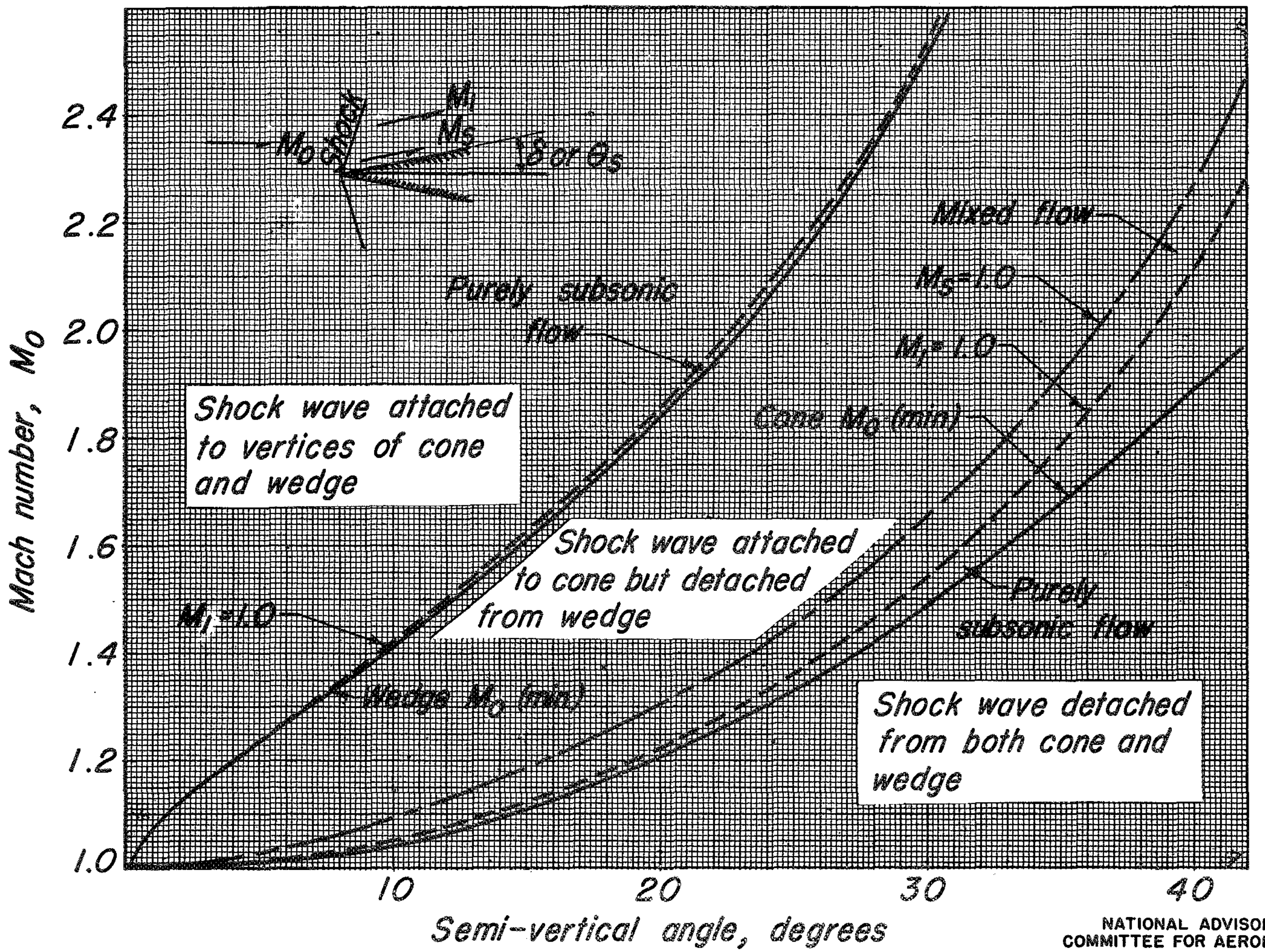

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Figure 2.- Characteristics of the flow about cones and wedges at supersonic velocities.

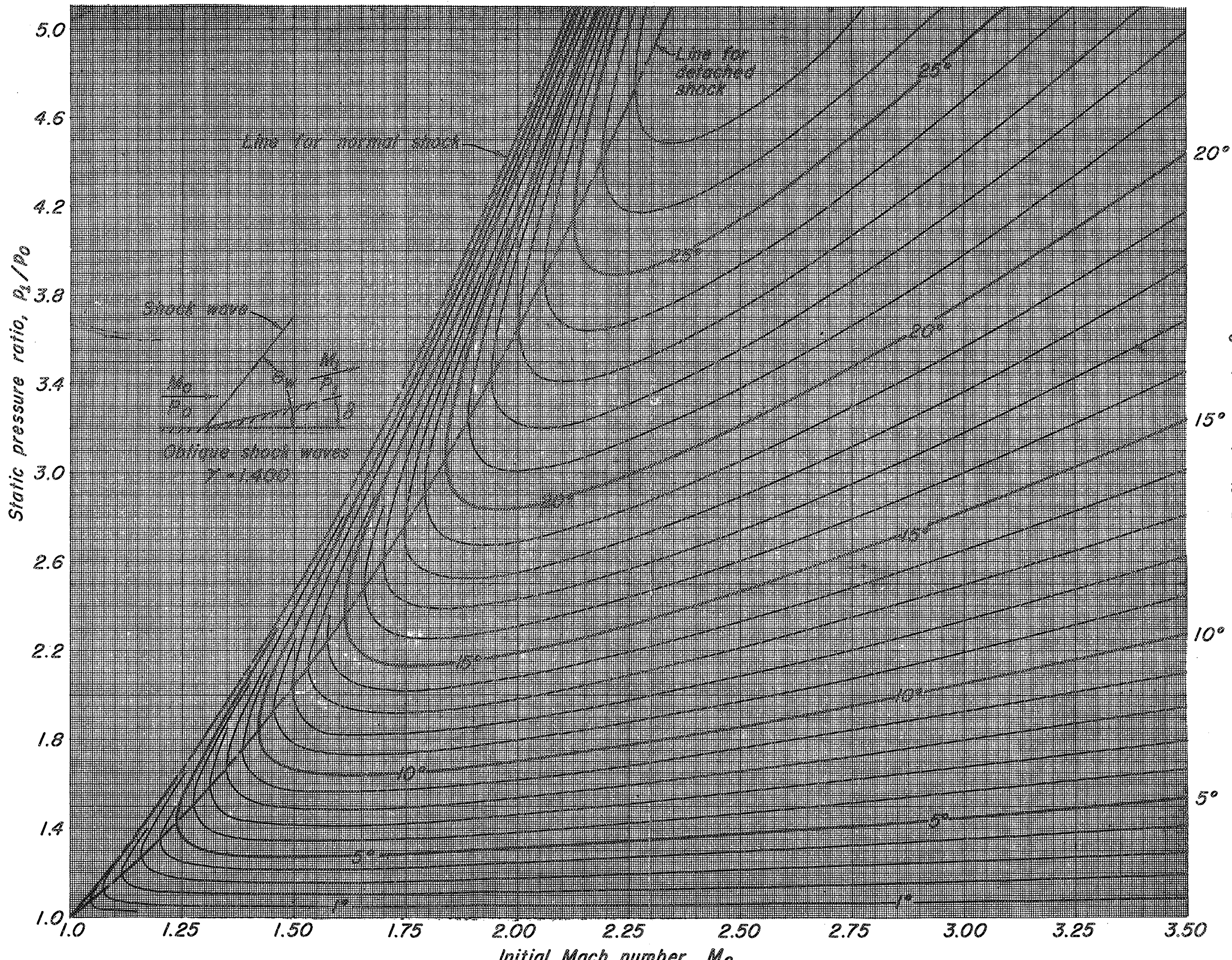

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Figure 3.- Variation of static pressure ratio with initial Mach number for various deflection angles.

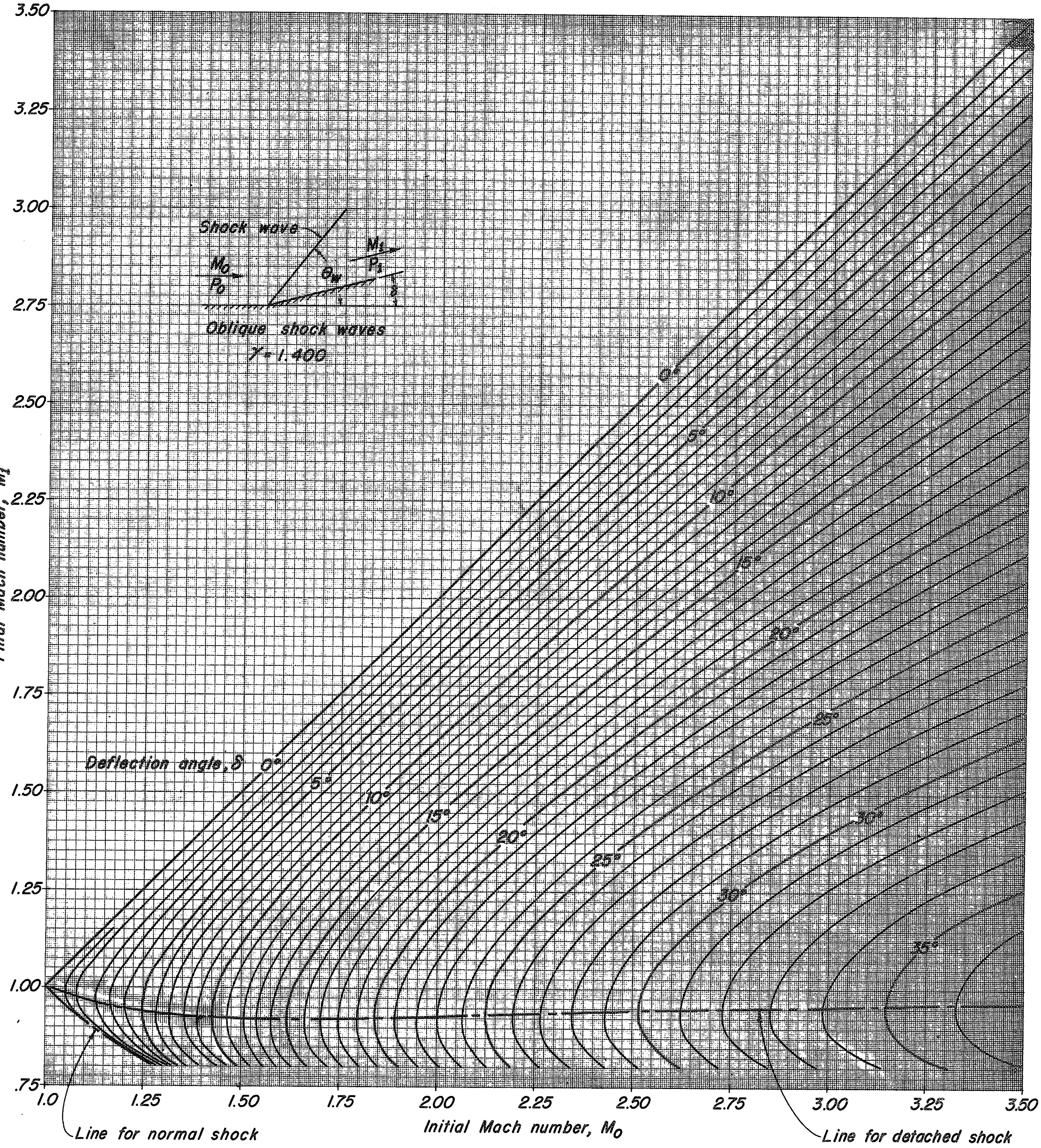

Figure 4.- Variation of final Mach number with initial Mach number for various deflection angles.

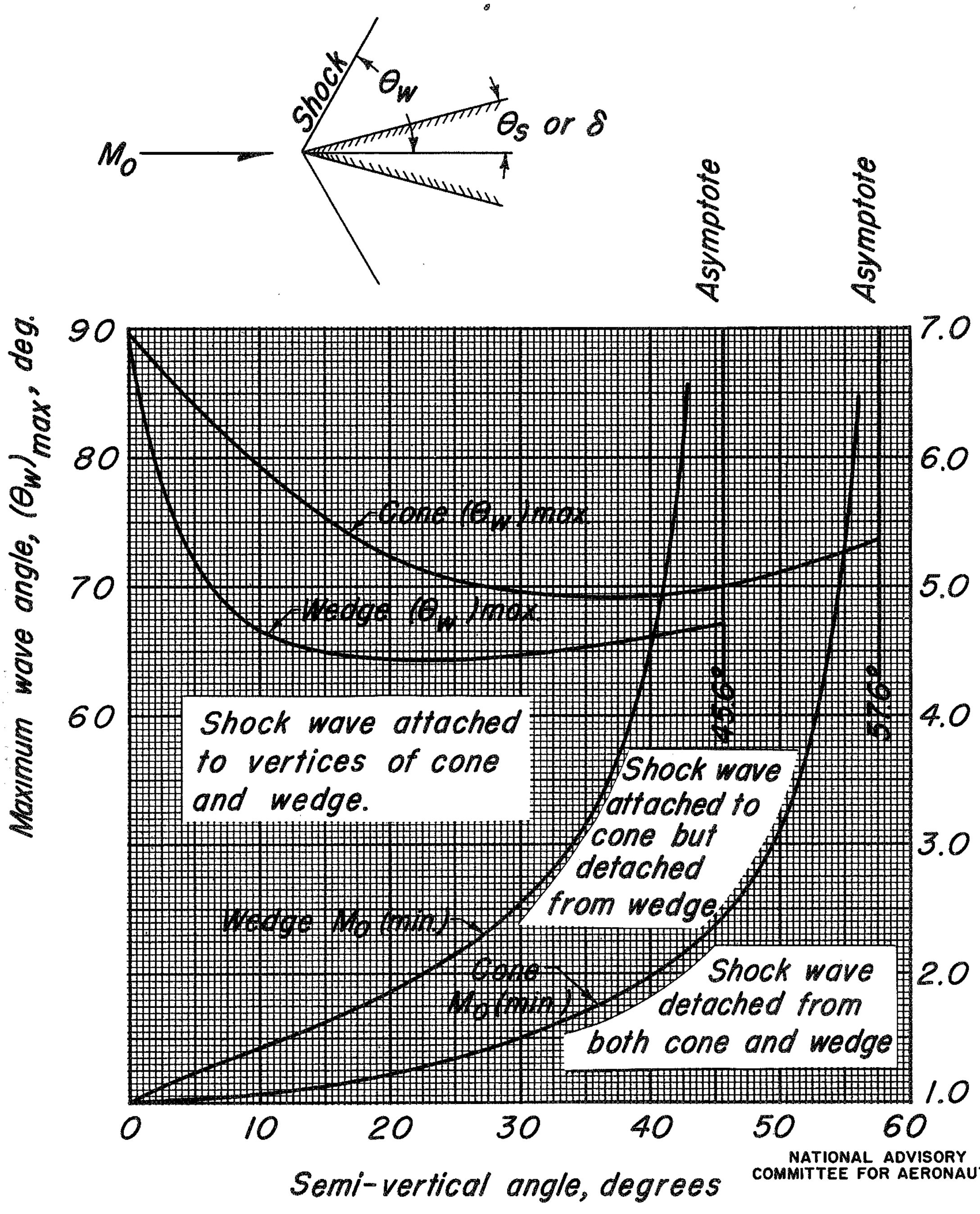

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Figure 5.- Critical conditions for detached shock waves on the vertices of cones and wedges.

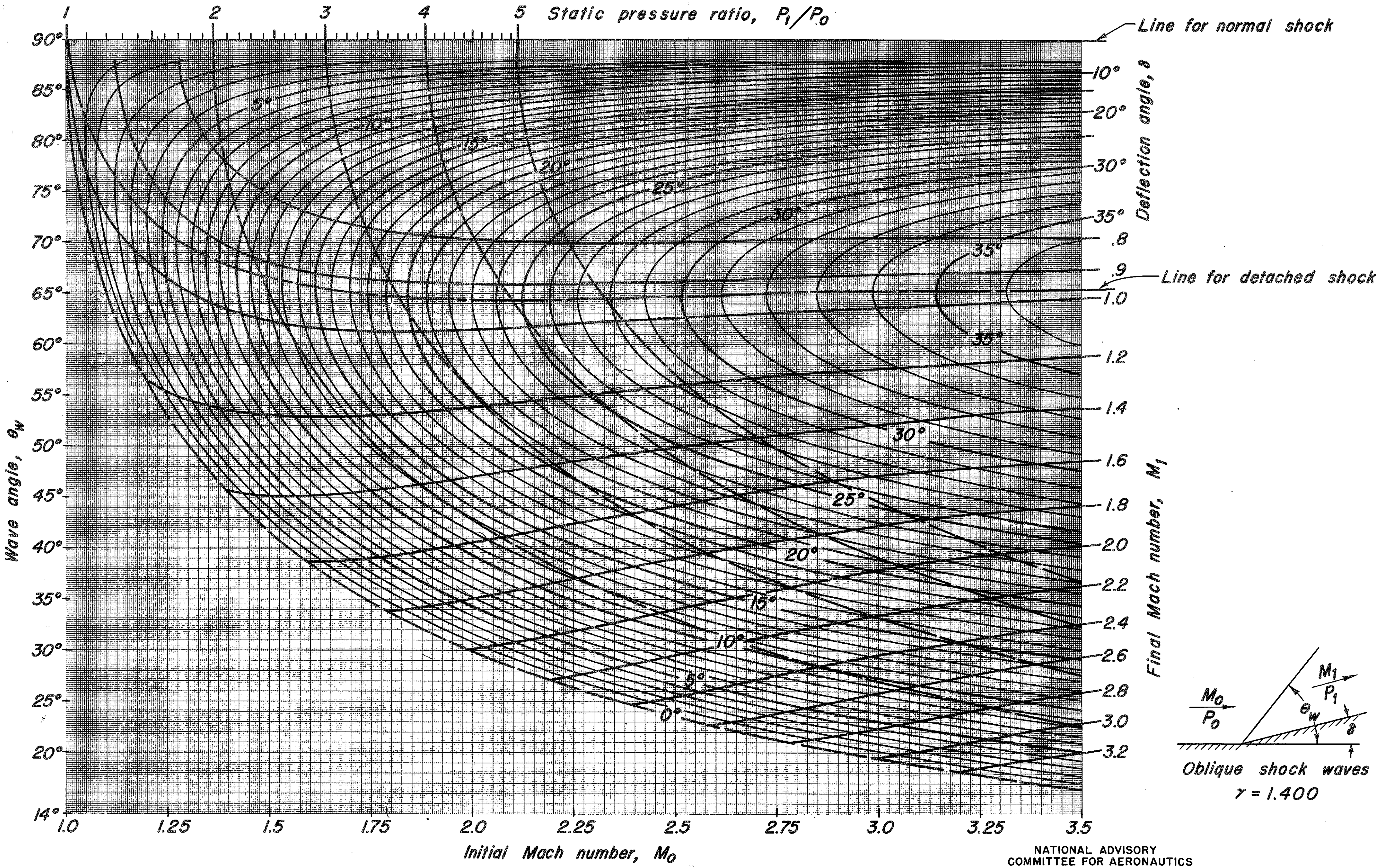
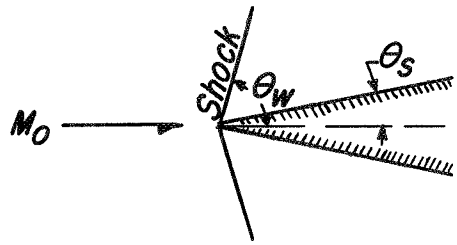


Figure 6.– Variation of wave angle with initial Mach number for various deflection angles, static pressure ratios, and final Mach numbers.



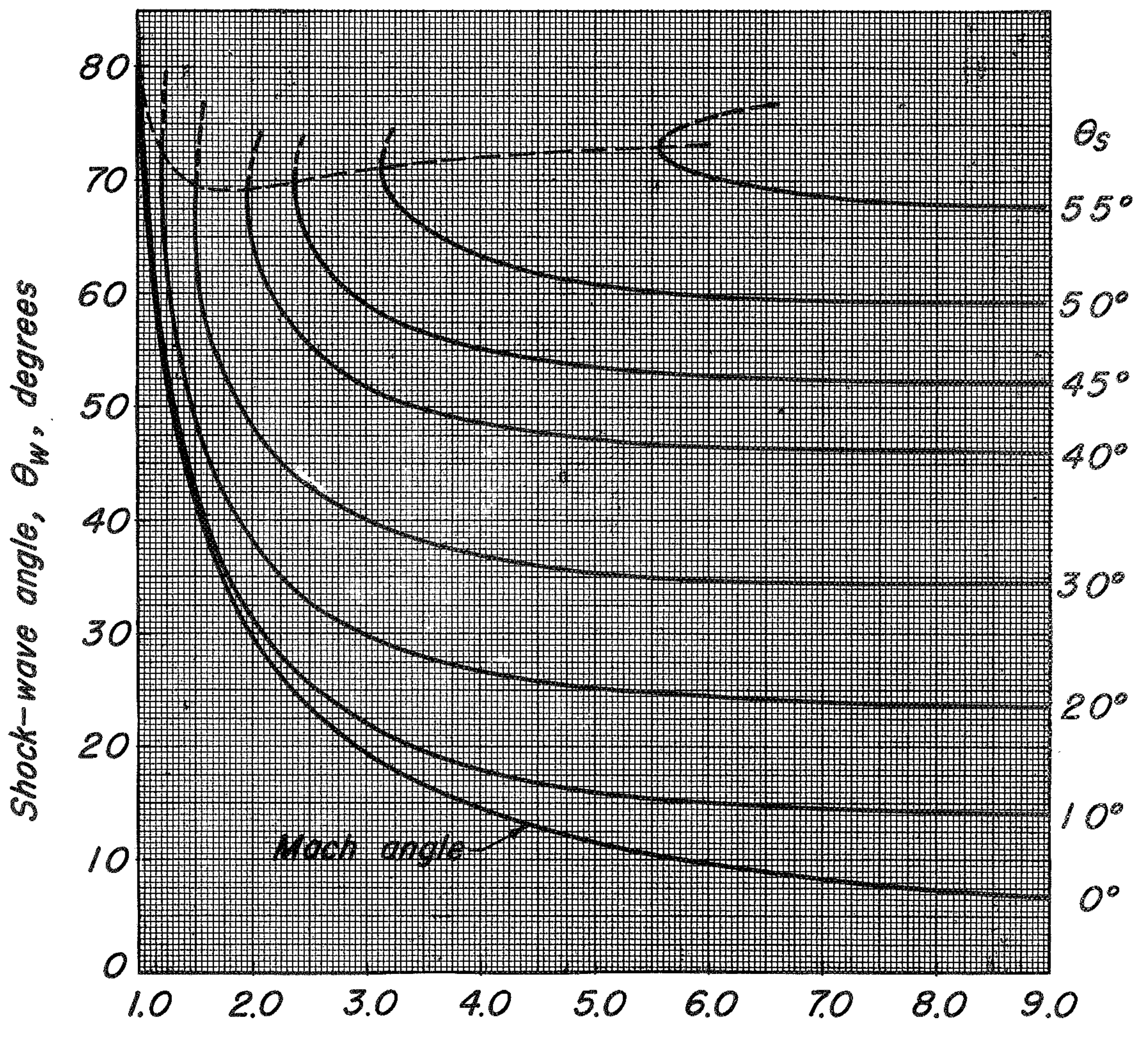


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Figure 7.- Variation of shock-wave angle with Mach number for various sized cones.

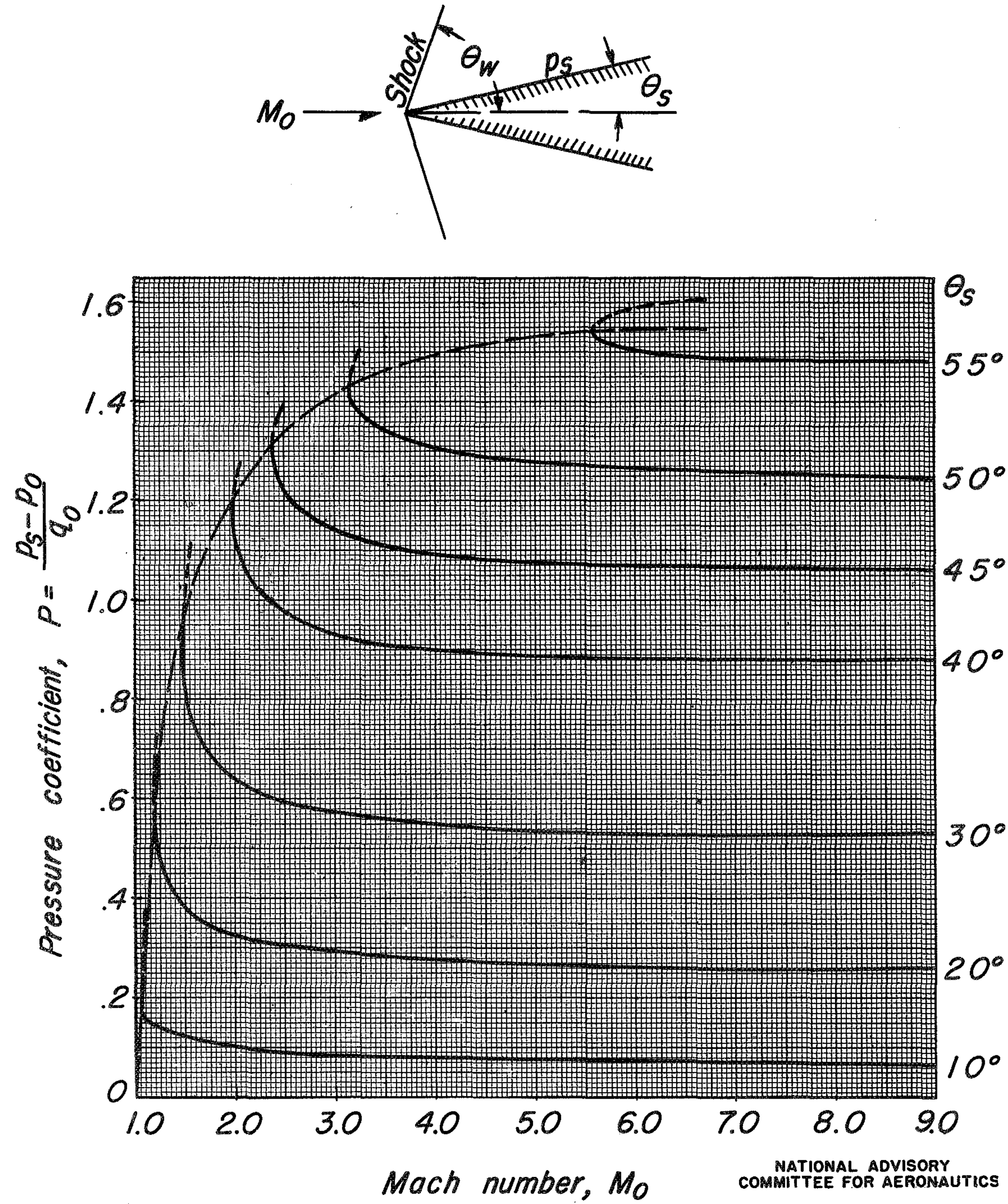

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Figure 8.- Variation of surface pressure coefficient with Mach number for various sized cones.

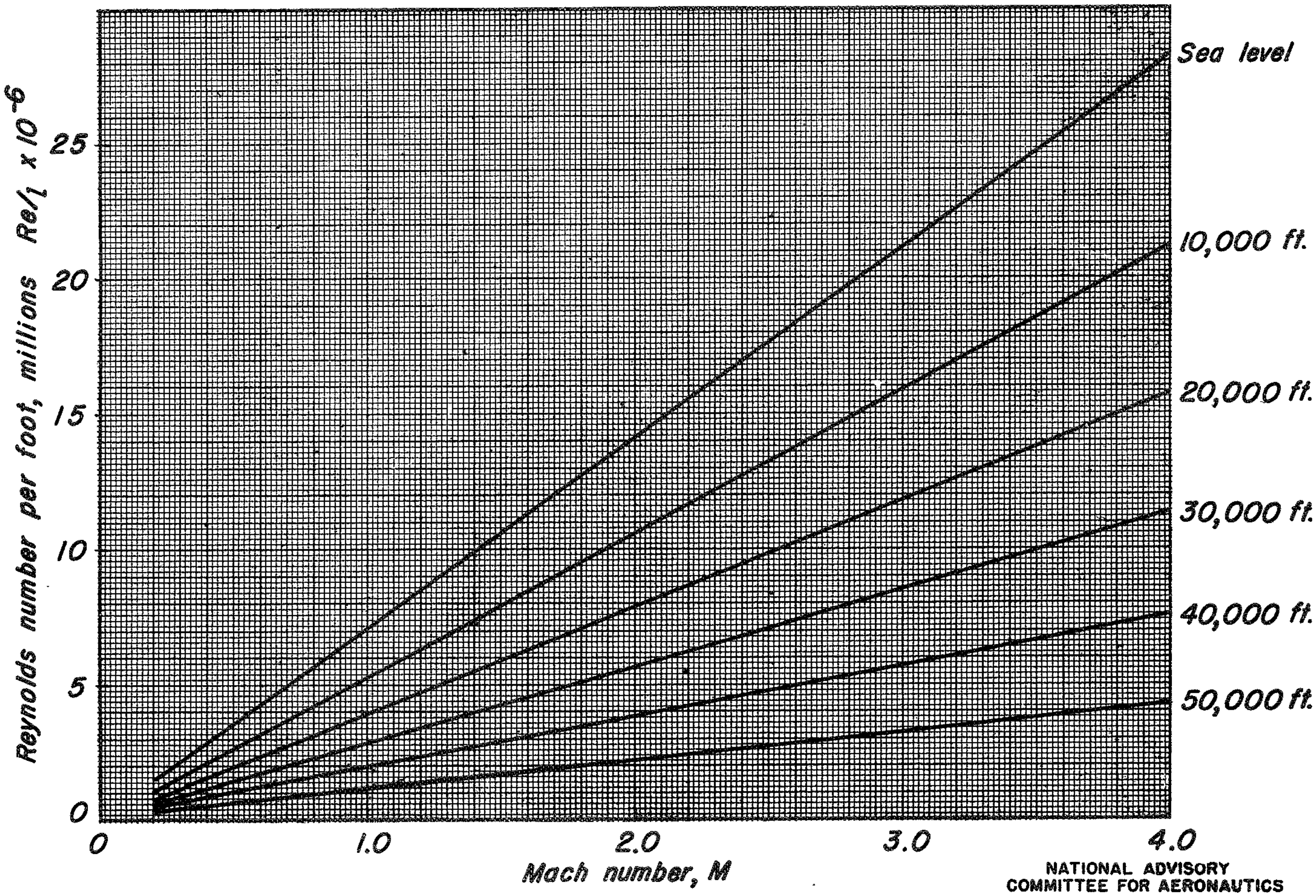
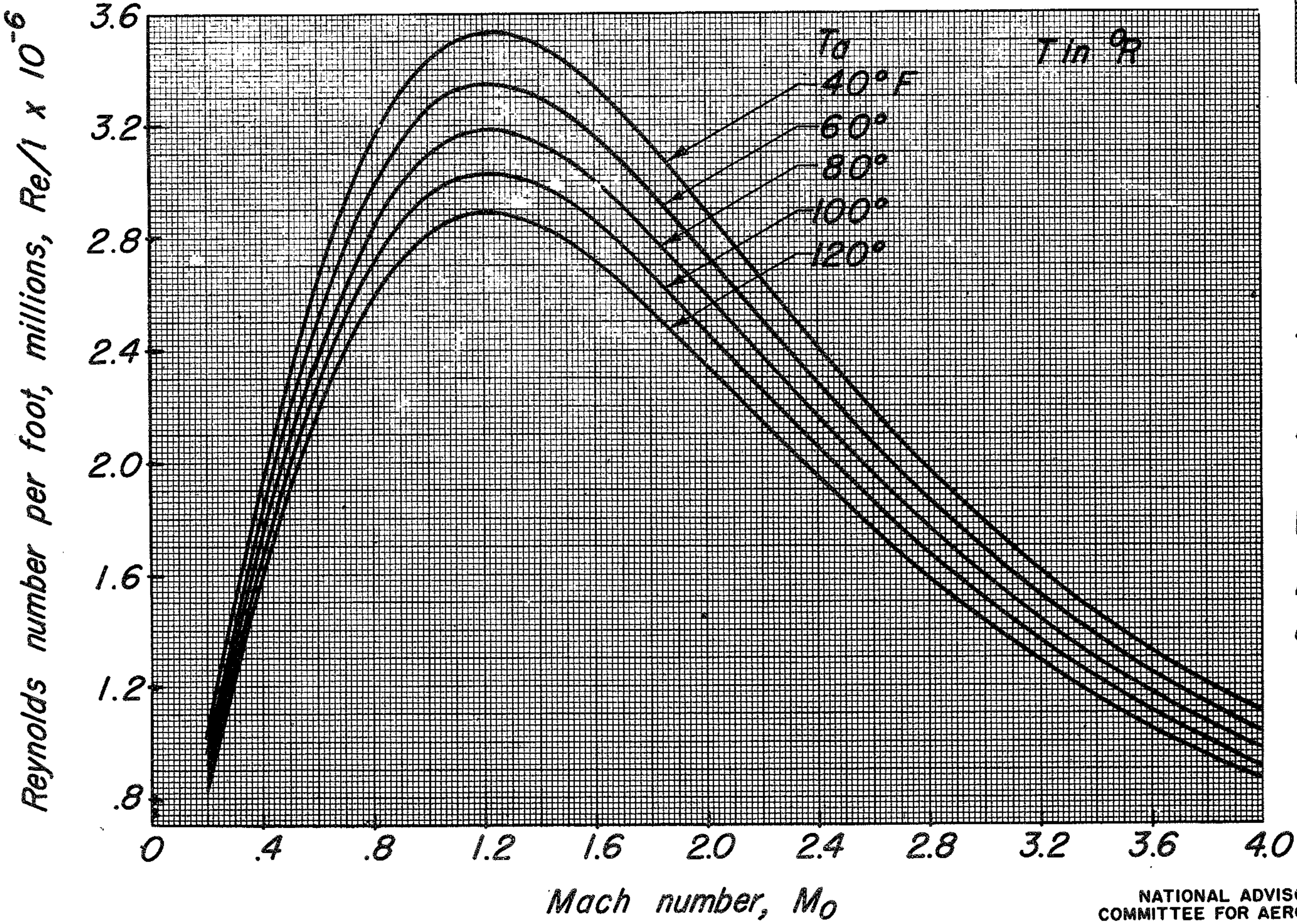
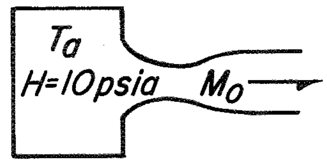


Figure 9.– Variation of Reynolds number with Mach number at various altitudes.

$$\frac{Re}{l} = \frac{\rho V}{\mu} = \frac{M_0 H}{\mu_a}\sqrt{\frac{\gamma}{RT_a}}\left(\frac{T}{T_a}\right)^{\frac{3}{2}}\frac{\left[\left(\frac{T}{T_a}\right)+\frac{216}{T_a}\right]}{\left[1+\frac{216}{T_a}\right]}$$





Air expanding isentropically from H=10psia.

Values of R_e/l in proportion for other values of H

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Figure 10.- Variation of Reynolds number with Mach number.